



Синергия освоения сложных систем и знаний как фактор роста креативного потенциала каждого школьника

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АННОТАЦИЯ

Введение. В современных условиях процессы управления школьным (профильные классы) математическим образованием на основе проявления синергии в освоении сложных систем и знаний в ходе поисковой и исследовательской деятельности каждого школьника являются актуальным направлением. Именно, на основе симбиоза математического и компьютерного моделирования актуализируются «проблемные зоны» и разрабатываются технологии адаптации к школьной математике обобщенных конструкторов сложных систем и знаний, в том числе современных достижений в науке (синергия цилиндра Шварца, элементы фрактальной геометрии, теория кодирования и шифрования информации, нечеткие множества и fuzzy-logic, теория распределений Л. Шварца, нелинейная и голоморфная динамика и т.п.) с эффектом роста креативного потенциала каждого школьника. Проблема исследования: какова синергия содержания и технологий адаптации сложных математических знаний к школьной математике с эффектом роста креативного потенциала в ходе поисковой и исследовательской деятельности каждого школьника. *Целью исследования* было разработать дидактические основы проявления синергии обучения математике сложного знания в школе с эффектом роста креативного потенциала в ходе поисковой и исследовательской деятельности каждого школьника.

Материалы и методы. Источниковой базой исследования является методология интеграции математического и компьютерного моделирования в ходе исследования обобщенных конструкторов сложных систем и знаний, метод параметризации и технологии наглядного моделирования математических объектов и фундирования опыта личности, компьютерный дизайн и вычислительные методы результатов исследовательской деятельности, методы диалога культур и вариативности модальностей восприятия и измерения роста креативного потенциала каждого школьника.

Результаты. Фактор-импульсом проявления синергии в обучении математике сложного знания является актуализация типологии модальностей восприятия и параметры роста креативного потенциала каждого школьника, определение этапов и технологии исследования сложных систем и знаний, анализ эталонов и новые результаты исследования «проблемной зоны» школьной математики (площадь поверхности и параметризация цилиндра Шварца).

Заключение. Предложенные инновации способствуют повышению эффективности методики развития креативности каждого школьника в поисковой и исследовательской деятельности и повышению качества освоения обучающимися математических конструкторов и процедур. Научная новизна результатов изучения многоэтапных математических и информационных задач адаптации современных научных знаний к школьной математике заключается в обосновании и технологии проявления синергии в обучении математике каждого школьника и актуализации технологии симбиоза математического и компьютерного моделирования.

КЛЮЧЕВЫЕ СЛОВА

синергия обучения математике в школе, сложные системы и знания, наглядное моделирование, фундирование опыта личности, современные достижения в науке, компьютерный дизайн

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Synergy of complex systems and knowledge mastering as a factor of creative potential growth of each school student

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ABSTRACT

Introduction. In modern conditions, the processes of school (specialized classes) mathematical education management based on synergy manifestation in the development of complex systems and knowledge during the search and research activities of each school student are a relevant area. Namely, on the basis of mathematical and computer modeling symbiosis, "problem areas" are updated and technologies are developed for generalized constructs adapting of complex systems and knowledge to school mathematics, including the modern achievements in science (synergy of Schwartz cylinder, elements of fractal geometry, theory of an information coding and encryption, fuzzy sets and fuzzy-logic, theory of L. Schwartz's distributions, nonlinear and holomorphic dynamics, etc.) with the effect of creative potential increasing of each school student. Research problem: what is the content and technologies adapting synergy of complex mathematical knowledge to school mathematics with the effect of creative potential increasing during the search and research activities of each school student. *The aim of the study* was to develop the didactic foundations for synergy manifestation in complex knowledge teaching in mathematics at school with the effect of creative potential increasing during search and research activities of each school student.

Materials and methods. The source base of research is the methodology of mathematical and computer modeling integration in the course of generalized constructs of complex systems and knowledge studying, method of parameterization and technology of mathematical objects visual modeling and founding of personal experience, computer design and computational methods of research results, methods of cultures dialogue and variability of perception modalities and growth of each student's creative potential measurement.

Results. The factor – impetus for synergy manifestation in complex knowledge teaching in mathematics is the actualization of perception modalities typology and growth factors parameters of each school student's creative potential, definition of technology stages of complex systems and knowledge studying, the analysis of standards and new results of "problem zone" studying at school mathematics (surface area and parameterization of Schwartz cylinder).

Conclusion. The proposed innovations contribute to an effectiveness increasing of an innovative methodology for developing the creativity of each school student in search and research activities and an improving the quality of school students' mastering of mathematical constructs and procedures. The scientific novelty of the results of multi-stage mathematical and information problems studying for modern scientific knowledge adapting to school mathematics lies in synergy justification and technology in teaching mathematics to each school student and an updating the technology of mathematical and computer modeling symbiosis.

KEYWORDS

synergy of mathematics teaching, complex systems and knowledge, visual modeling, personal experience founding, modern achievement in science, mathematical and computer design

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INTRODUCTION

UNESCO coordinates the implementation of "Education 2030" agenda to reimagine education for a sustainable future and to educate every learner, provide the quality learning opportunities for all, and empower them to be a creative and responsible global citizens. The nonlinear trends increasing in the world around us, situationality and uncertainty in individual decision-making, the need for choice ambiguity in problematic situations create the uncertainty and the entropy increasing in educational processes (Pei – Shan Hsieh [25] – opportunities of virtual reality, Tongxin Li [24] – embedding uncertain temporal knowledge graphs, R.I. Semenova [21] – STEAM-education and IT-employment as factors of adaptation) The global trend towards a drop in the levels of educational and professional motivation of school students, the tendency to "simplify" the content of mathematics, moving away from abstraction and complexity, the main focus of education on problems solving of personality socialization leads to an increase in crisis phenomena in mathematical education, both at school and at university. Moreover, the penetration of post-non-classical rationality paradigm in the process of teaching mathematics becomes almost the only opportunity to lay an effective foundation for thinking reform, which is being talked about by foreign and Russian scientists (R.R. Maaliw III [9] – adaptive virtual learning environment based on learning styles for personalizing e-learning system, A.I. Kreyk [8] – synergy aspects of organic modernization, S. Fatahi [4] – an experimental study of an adaptive e-learning environment). Namely, the inclusion of each school student in a level-based complex activity is able to solve the problems of self-organization and self-development of a personality in the process of teaching mathematics, dictates the need to include motivational-value and emotional-volitional, research and metacognitive, social and personal behavioral strategies in the course of subject content mastering into a single integrity and creative potential growth (J. Santos [18] – innovative pedagogical practices, S.N. Dvoryatkina [26] – technological stages of Schwartz cylinder's computer and mathematics design using intelligent system, Po Ling Chen [28] – cognitive and social-emotional skills training). It is the management of educational processes based on the development of complex knowledge by means of mathematical and computer modeling that can give a powerful motivational charge to the study of mathematical disciplines; as a result, there will be increased the interest in mastering mathematics with the real development of theoretical and empirical thinking (comparison, analogy, analysis, synthesis, etc.), the processes of dominance of both logical and probabilistic reasoning schemes are realized, clear dynamics of the course of reasoning and fuzzy logic, the ability to learn and highlight the main things, the ability to theoretical and empirical generalization, analysis, synthesis, computer and mathematical modeling. At the same time, the possibility of modern achievements adapting in science to school mathematics and computer interactive interaction with an educational subject enhances the developmental effect and increases educational motivation, reveals the connections with real life and practice, creates the phenomenon of synergetic effects in the development of complex mathematical knowledge in an open and saturated information and educational environment (E.I. Smirnov [29] – manifestation of synergy in the study of multy-stage mathematical and information tasks, E.V. Soboleva [30] – development of critical thinking through the involvement of students in competitive robotics). A school student should already show a synergetic style of thinking in mastering of complex knowledge generalized constructs, know and find the associations in real life of such phenomena of collective orderliness as the Jabotinsky-Belousov effect, Benard cells, Lotka-Volterra equations in the predator-prey system, Koch snowflake and Schwartz cylinder, the scenario of P. Ferhulst and constants of M. Feigenbaum, the strange Lorentz attractor, etc.

The leading idea: key aspect of each school student's creative potential growth can be the mode of synergetic effects manifestation in teaching mathematics with the possibility of updating the mastering stages of mathematics "problem zones" and the adapting of complex knowledge generalized constructs to school mathematics by means of mathematical and computer modeling, the identifying attributes of cognitive activity and social interactions self-organization (attractors, fluctuations, bifurcation points, pools of attraction, iterative procedures, etc.) in an open information and educational environment. At the same time, it is necessary to reasonably use computer systems, platforms and pedagogical products in mathematical education (MathLab, MatCad, GeoGebra, Qt-Creator, Maple, etc.) that support the development of complex mathematical knowledge based on visual modeling and dialogue of mathematical, information, natural science and humanitarian cultures, up to the implementation of artificial intelligence systems. Thus, this study is an attempt

to develop a technology for synergy manifestation and modern achievements in science adaptation to school mathematics based on computer modeling and design, visual and mathematical modeling of complex systems and knowledge in the "problem zones" of mathematical education with the manifestation of synergetic effects and each school student's creative potential growth.

METHODOLOGY, METHODS AND TECHNOLOGIES

1. The design and implementation of synergy manifestation in the development of complex knowledge generalized constructs are associated with the development by school students the level and stage-by-stage application of mathematical and computer modeling tools in an open and saturated information and educational environment. An effective tool for complex mathematical knowledge mastering, growth of creative potential and an intellectual thinking operations development of each school student can be the study and adaptation to school mathematics of modern achievements in science, vividly and meaningfully presented in applications to real life, development of other sciences, high technologies and industries. At the end of XX and the beginning of XXI centuries, the paradigm of post-non-classical rationality was developed in the representation of scientific picture of the world as a set of theoretical and methodological attitudes based on the priority of self-organization processes of dynamic nonlinear systems in conditions far from equilibrium based on coordinated actions of various factors (G. Haken [6] – synergetic approach to brain activity, G.G. Malinetsky [10] – nonlinear dynamics of scientific picture of the world, K. Mainzer [11] – complex dynamics of thinking, B. Mandelbrot [12] – fractal geometry of self-organization processes, I.R. Prigozhin [17] – nonequilibrium statistical mechanics, etc.). Therefore, the need for synergy manifestation of mathematical education is additionally conditioned by the possibility of school student's involvement in the processes of self-organization and motivation to master the modern achievements in science. The researches of I.V. Abramova [1] – Innovative aspects of the organization of design and research activities, Rui Luo [5] – model identification based optimal control, Tongxin Li [24] – embedding uncertain temporal knowledge graphs, Hyunsuk Han [27] – operating characteristic curve in educational assessment, Po Ling Chen [28] – cognitive and social-emotional skills training, Pei-Shan Hsieh [25] – virtual reality and children's hope, pleasure, and travel inspection, lity of implementing onditions for improving the effectiveness of mathematical education and personal development based on the adaptation of modern achievements in science to school mathematics: information saturation of motivational field of teaching (including processes of school digitalization), multiplicity of goal setting and the search for bifurcation transitions in mathematical activity, fluctuation diversity of parameterization and integration of mathematical, informational, natural science and humanitarian knowledge in the construction of mathematical results in the form of an attractors and pools of attraction of nonlinear transformations based on mathematical and computer modeling (including the possibilities of neural networks and intelligent systems using). There are opportunities to manage of school mathematics education with the manifestation of synergetic effects and there are prospects for an adequate response to modern challenges and contradictions in mathematics education that meet the needs of personal development and creative potential growth of each school student.

The **synergy of mathematical education** will be considered by as a symbiosis and a qualitative change in the nonlinear effects of self-organization and self-development of personality during the development of mathematical activity in the context of complex stochastic processes managing based on the coordination of different factors and principles in three contexts: meaningful (semiotic), procedural (imitation) and social (E.I. Smirnov [23]). An important context is the formulation of external factors of influence in the form of a multiplicity of goal-setting, building stages and hierarchies of sign-symbolic and figurative-geometric activities in the direction of essence founding of mathematical objects and procedures, searching and analyzing side solutions using the information technology, an identifying bifurcation transitions and basins of attraction in the studied processes based on cultures dialogue, variability and parameterization, ensuring coherence information flows in the emergence of new product (at the same time in terms of network interaction) (K. Mainzer [11]). In the article of S.N. Dvoryatkina [2], all stages of the manifestation of mathematical education synergy are identified and characterized: preparatory, content-technological, evaluative-correctional and generalizing-transformative.

The following is a detailed description and characteristics of deployment sequence of manifestation stages of mathematical education synergy:

- *Diagnostics of perception modalities* (sign-symbolic, figurative-geometric, verbal (historical-genetic context), concrete activity (computational and algorithmic modeling), information (computer modeling and design, programming, experiment)) of representative systems and determining the school student's priorities in search and research activities; diagnostics of synergetic effects (including creative potential) and the present state of personal meanings and preferences in mastering ways of mathematical content.
- Defining the *principles of "problem zones" identifying* and selecting criteria for volume, structure and content of generalized constructs in mathematical knowledge development that have the potential for complexity and opportunities for synergy revealing in teaching mathematics. It is realized the context of an emotional response to an applied effect, updating visual and digital models and taking into account of school student's personal preferences.
- Presentation of *generalized constructs depository of modern achievements in science*, analysis and selection of mathematical and computer modeling methods, description of historiogenesis and applied effects, "problem zones" of research and possibilities to school mathematics adaptation (presentations, abstracts, lectures, computer design). Presentation of samples of scientific problems (at the reference and situational levels) with synergy manifestation of complex knowledge by mathematical means based on the ICT implementation tools to mathematical education support.
- *Selection of practice-oriented tasks content* in research niche of complex knowledge generalized constructs, design of stages and forms, means and technologies for specific problems adapting of modern achievements in science for small groups of school students. It is based on visual modeling and founding of personal experience (pedagogical support, samples of technological approaches, data collection, thematic plans and scenarios of multi-stage mathematical and informational tasks, distribution of roles in small groups).
- *Multiple goal-setting* to identify the essence of cognitive practice-oriented tasks and actualization of perception modalities in founding and study of complex systems and knowledge generalized constructs; informatization and digitalization of search and research activities, actualization of techniques and methods of experimental mathematics methods mastering in the "problem zones" of teaching mathematics; symbiosis of mathematical and computer modeling in the interpretation of results; actualization of attributes and parameters of synergy manifestation of scientific problem (the "problem zone" of mathematical education) with details, analysis, features and stages (attractors, bifurcation points, fluctuations, pools of attraction, etc.).
- *Actualization, generalization and evaluation* of mathematical, informational, humanitarian and natural science knowledge and methods in the procedural period of "problem zone" studying in the manifestation's context of integration, phasing and variability. Criticality and creativity in obtaining by-products of research (identification of patterns, laboratory calculation classes, multi-stage mathematical and informational tasks, search for pools of attraction, attractors and iterative procedures, bifurcation transitions and determination of fluctuation dynamics).

First of all, it is necessary to search and study the "problem zones" of mastering the mathematics of complex knowledge with the actualization of universal educational actions (UMS) and the integrity of complexes of practice-oriented tasks perception accompanying of complex knowledge generalized constructs in meaningful, personal and social contexts. According to E.I. Smirnov [23], the "problem zones" of mathematical education is a complex of substantive, procedural and personality – adaptive components of teaching mathematics, based on the disclosure of contradictions and cognitive activity problems in specific area and aimed at finding and exploring the essences of its complex educational elements.

At the same time, the principles of identifying "problem zones" in teaching mathematics are determined (with the actualization of modern achievements in science):

- Mathematical tools and methods for problems solving should be in the field of actual experience and representative systems of each school student's personality (meaningful lines of school

mathematics, variability of tasks, ontologies of generalized constructs, funding chains of tasks, perception modalities, etc.).

- The process of identifying the essence of educational elements in complex "problem zones" is based on the multiplicity of goal setting and the possibility of mathematical and computer modeling of a generalized constructs (surface area, Ferhulst scenario, Schwartz cylinder, limit of function, Cantor set, tiling of the plane, etc.).

- Various search samples are possible using the information technologies (computer modeling, maple leaf by M. Barnsley, cross sections of bodies, number n by the Monte Carlo method, the area of Schwarz cylinder polyhedral surfaces, etc.).

- The presence of novelty and entertainment elements (Dirac δ -function, Sierpinski napkin, fuzzy sets, Poincare model, Caesar cipher, Schwarz cylinder, etc.).

The revealed principles allow the teacher to rely on the criteria of complex systems and knowledge selection for the organization of effective search and research activities of each school student:

- The beauty and harmony of mathematical formulas and procedures with synergetic effects and an essence manifestation of complex knowledge generalized constructs (modern achievements in science) in the form of visual and digital models of an essence representation.

- The effectiveness and accessibility of an adaptation processes of complex knowledge generalized constructs and results achievement (meaningful, procedural and personal) of research activities by each school student.

- The possibility and necessity of "problem zones" of mathematics studying by using of an information media and digital platforms in a saturated information and educational environment.

- A significant and accessible manifestation of studying applied effects of complex problem and emotional response to them by means of mathematical and computer modeling; an ability to effectively solve on problems of science, life, communication, technology and production.

- The possibility of dialogue cultures and coherence of school student interactions in the process of cognitive activity, taking into account of personal preferences.

- Variability and actualization of perception modalities (sign, verbal, figurative, geometric, concrete-activity, digital) and a mutual transitions of sign systems in the development of mathematics complex knowledge.

School mathematics should be oriented and motivated to integrate of science and education, to adapt the modern achievements in science (complex knowledge) to the actualization of "problem zones" of mathematics development, creating an effective motivational field of learning, to the ability to "launch" the processes of self-organization and synergy in teaching mathematics, to improve the quality of mathematics development, including mathematical literacy and school students' creative potential growth.

2. Researchers of various sciences (G.G. Malinetsky [10] – synergy theory, K. Mainzer [11] – self-organization theory, A.N. Poddyakov [16] – complexity theory, etc.) convincingly showed that an effective personality development occurs when complex knowledge mastering (different levels of its complexity depending on school student's personal development), creating the situations overcoming difficulties in the process of knowledge mastering and creating a unified picture of the world based on a high degree of school student's educational motivation deployment in a single network of interactions, independence and coherence. In cognition of the complexity, the process of cognition itself "becomes a communication, a loop between cognition (phenomenon, object) and cognition of this cognition" (E. Morin [14]). Together with A.N. Poddyakov [16], we note the following features in solving complex problems in mathematical education:

- In the behavior and development of a complex dynamic system such as mathematical education, there is always a share of uncertainty and unpredictability; it requires a variety of different

descriptions and solutions both in content and in cognitive processes that differ from each other and complement each other; no less an effective tools are concepts that are not strict and fuzzy, built on the basis of an empirical rather than theoretical generalizations, the study of which is impossible without of computer and mathematical modeling using (S. Fatahi [4]).

- The complex system of educational elements mastering is characterized by changes not only at the level of specific manifestations, but also at the level of its essence (generalized constructs), which is most significant for updating the processes of understanding and the presence of self-organization developmental effects. In complex educational systems, an effective rule (founding modes (E.I. Smirnov [22]) of phased deployment of the entity can be distinguished (including method of inverse problems of self-organization theory (G.G. Malinetsky [10])), but they will inevitably be a quite variable in self-organization types based on the implementation of visual modeling (E.I. Smirnov [22]) and fundamentally have the depending context.

- Theoretical models of high level arbitrarily are fundamentally limited (D. Meadows' "limits of growth"). For effective research of complex dynamic systems, various search samples are needed (experimental cross-sections, comparative analysis of specific manifestations, computer modeling, analogies, analysis through synthesis, etc.) – real interactions with the system based on visual modeling, and not only theoretical activity with its abstract models.

- When studying a complex system, variability and multiplicity of goal setting are necessary the setting of diverse and multi-level goals (multiple goal setting) that can compete with each other. One of the main emotional states of a person in the study of complex systems in mathematical education is uncertainty, doubt, willingness to accept a two-fold (based on prediction and random) results of actions, etc.

- The results of personal activity with a complex system of content and methods of mathematical education, the results of interaction with it cannot be predicted completely, exhaustively. Probabilistically guaranteed educational outcomes are possible. Moreover, along with direct, predictable educational results, a variety of side effects, unpredictable products of personal development and mathematical activity are formed.

Managing the synergy of mathematical education in schools and universities is possible and effective when characterizing, building hierarchies of order and updating internal attributes (mechanisms) of various types self-organization (meaningful – figurative-geometric, sign-symbolic, verbal, concrete-activity; procedural – clusters and spirals of the foundation of personal experience, visual modeling methods, technologies, techniques, ideas, algorithms of cognitive activity, personal adaptation and social – dialogue of cultures, visual modeling of the foundation modes, actualization of personal meanings and preferences, communication in small groups, choice of forms and methods of cognitive activity based on variability and diagnostic procedures, value orientations, etc.) (S.N. Dvoryatkina [3]). The research and an importance of self-organization examples in living and inanimate nature through the processes of destruction and creation (chaos and order) have shown that complexity increase in open and non-equilibrium systems is not a destructive mechanism, but on the contrary is a necessary transition to a new level of development, more complex and orderly forms of organization, including in educational structures. In school student's mathematical education, the most important problem is the issue of adequate perception of educational material and activities leading to understanding against the background of a high educational or professional motivation.

Let's highlight the following system-genetic contexts of mathematical education synergy, both at school and at university.

- **Procedural contexts.** The basic construct of presented idea of modern achievements in science adaptation is the concept of personal experience founding. What is the phenomenon of founding? Founding (German: fundierung – justification, foundation) is a term used in phenomenology (and in other sciences) to describe the relationship of ontological justification. E. Husserl defines the relationship of founding as follows (E.I. Smirnov [22]): A is founded by means of B if B is essential for the existence of A, only in unity with which A may exist. The founding relationship can be one-sided (A is founded in B) or two-sided (A and B are founded in each other). According to the

phenomenological doctrine, all complex high-level acts and objects are based on the original simple acts and objects. For the first time, the concept of founding was introduced into pedagogy by V.D. Shadrikov and E.I. Smirnov in 2002 (E.I. Smirnov [22]) as a process of creating conditions for the gradual deepening and expansion of school knowledge in the direction of mastering the complex knowledge generalized construct, as a process of personal choice, the development of intelligence and creativity of the individual. Therefore, the concept of the personal experience founding acts as an effective mechanism for overcoming educational and creative crises of individual formation and actualization of integrative links between science and education. Technologically, the founding is carried out on the basis of identifying mechanisms and conditions (psychological, pedagogical, organizational, methodological, material and technical) for updating the school student's personal preferences in the study of "problem zones", followed by theoretical generalization and expansion of complex knowledge content (S.N. Dvoryatkina [26]).

In educational psychology, a number of conditions have been identified that contribute to the creative activity of school students, the formation of intellectual operations and universal educational actions. Therefore, we propose the following pedagogical conditions for the formation of cognitive and creative synergetic effects in an innovation process of (E.I. Smirnov [22]):

- *Presence of a creative environment* (stimulating a situation of success; tolerance to uncertainty; willingness to discuss and multiple solutions to the problem; identification and popularization of creative behavior patterns and its results).
- *Low degree of behavior regulation* and an availability of subject-information enrichment.
- *Information technology support and computer design* of school students' mathematical activities at all revealing stages of synergetic effects manifestation and the essence of "problem zones".

Complex knowledge mastering in conditions of uncertainty and unpredictability of research ways and methods requires the implementation of multiple goal-setting processes at cognitive activity with the possibility of visual models and empirical data arrays obtaining for computer means analysis. These can be: the cross-platform Qt Creator environment for the studying of Schwartz's cylinder (or "boot") (H.A. Schwartz [19]), computer algebra systems MathCad, MathLab, Maple, Mathematica, etc., – for the studying of stability solutions problems to differential equations (S.N. Dvoryatkina [3]), dynamic geometry systems Geogebra, Autograph, Living Geometry etc. – for the construction and studying of fractal objects (Koch snowflake, Mandelbrot and Julia sets, Cantor dust, etc.), identification of an attraction pools and attractors of iterative processes (Sierpinski napkin, maple leaf M. Barnsley – V.S. Sekovanov, Julia sets (V.S. Sekovanov [20]), etc.). At the same time, the processes of motivational field creating for the studying of complex mathematical constructs require the computer design and visual modeling of modern achievements in science (the strange Lorentz attractor, fuzzy sets and fuzzy-logic, Menger sponge, Ferhulst scenario (V.N. Ostashkov [15]), etc.). Building hierarchies in the deployment of generalized "problem zones" constructs essence based on the parameterization and abstraction, searching for bifurcation points and basins of attraction by means of building iterative processes. Also, the information technology support creates the mechanisms for complex knowledge adaptation to school and university mathematics (S.A. Tikhomirov [29]).

The processes of visual modeling in teaching mathematics as a concept, theory, method and technology were developed by E.I. Smirnov [22] and are designed to solve the tasks of school student's personality an actively involving in the cognitive process based on modeling of complex objects, procedures and situations in mathematical education, leading to the formation and development of an intellectual operation of "understanding" and an identification of progress stages towards the essence of mathematical education "problem zones".

The practice of comprehending the essence of a subject (mathematical object or procedure) by a school student in certain categorical field of knowledge and methods activity, sufficient for the success and effectiveness of operating with it, does not necessarily coincide with the objective content and expression of subject's an essential connection necessary. Moreover, it is possible to attach the additional links, which together with the necessary links create the integrity and hierarchy

of categorical field entity. This variability and mobility of an object essence requires the actualization of a step-by-step progress towards its cognition, causes a third dimension of the essence in its characteristics (personal adaptation) and defines the three-component integrity of the of an object essence as an object of cognition in cognitive activity course. Thus, we present the following structural and functional model of the essence for mathematical learning elements (Fig.1).

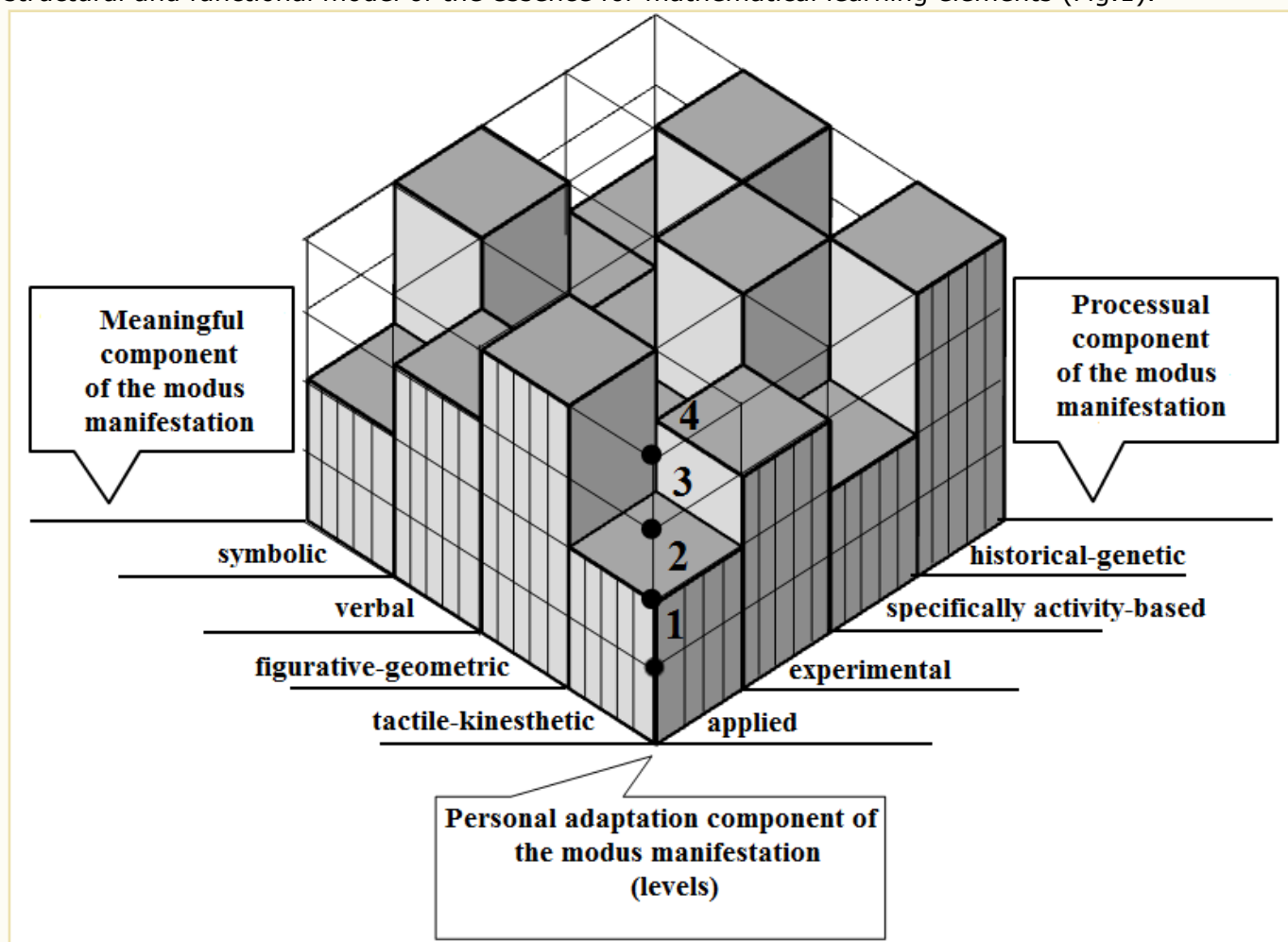


Figure 1 Structural and functional model of the essence of mathematical learning elements

- *The personal adaptation and social context of synergy manifestation in mathematical education.* Knowledge and values that are mediated in the process of teaching mathematics can be accepted and can become the property of the school student when they are actively processed and assimilated not by an individual, but become the communication content and activity in a group if they are integrated into the totality of all the information that the group has. When organizing group creative activity, it is necessary to create the conditions for generating a multiplicity of solutions to the problem based on information enrichment, intellectual tension and a low degree of behavior regulation. Thus, in the group form of work, school students have the opportunity to show the supra-situational activity and an implement technique for activating of creative thinking in mutual dependence, an actualizing the dynamics of creative process (intuition, verbalization, visual modeling, formalization, reflection, verification) based on the synthesis of convergent and divergent thinking.

The type of generalized construct modeling of modern scientific knowledge based on the revealed essence can be phenomenological and genetic. Following the theory of V.V. Davydov and D.B. Elkonin, it can be noted that the *phenomenological type* corresponds to the attributes and properties of an empirical thinking formation, when there is a designation of sensually given properties of objects and their connections, an abstraction of these properties, combining them into classes and generalization based on the formal identity of their individual properties and their external changes in interaction.

The *genetic type* of modeling corresponds to the attributes and properties of theoretical thinking formation, when the establishment of object's implicit hidden essential connections processes and phenomena of the role and functions of components relationship. So, within the system, the conditions of their origin and transformation are carried out. Therefore, the technology of synergy manifestation in an adaptation process of modern achievements in science in school mathematics can be oriented by accordingly to the phenomenological or genetic type of essence revealing for scientific knowledge generalized construct.

The founding procedures for the transition from the present state of the entity and its actual representation to the generalized potential development of the entity in the form of an ideal object (process or phenomenon, state of personal qualities) are multi-stage, multifunctional, directional and integrative in terms of intra and intersubject relationships actualization. At the same time, the transition procedures of an immediate development "zones" will be more pronounced and directed if the indicative and informational foundations of trainee's educational activities are cemented by learning content specially designed, visually modeled in the form of founding spirals or clusters of basic educational elements (E.I. Smirnov [22]).

RESULTS

The technology of identifying and researching "zones of modern achievements in science" in relation to teaching mathematics allows you to design and implement the stages of modern achievements in science adaptation to current state of school student's experience in mathematical activity. It allows you to integrate knowledge from various fields of science in the context of school mathematics, creates a precedent for school student's research activities when working in project activities (including, in small groups) and in the form of school student's an individual educational routes deployment, actualizes the synergistic effects in the process of complex knowledge mastering.

We will highlight a number of *technological stages of founding procedures deployment* in the processes of modern scientific knowledge adaptation to school mathematics with synergetic effects manifestation and modeling reflection of phenomenological type of generalized construct essence:

- motivational (self-actualization ("I'm interested in this")) – manifests itself in the severity of value and personality – adaptive characteristics of trainees cognitive activity in mastering the standards and samples of visual modeling phenomenology of a generalized construct and the results of diagnostic procedures: the significance and value orientations, the choice of ways of activity to disclose a separate quality of generalized essence manifestation (meaningful or procedural component – see Fig. 1); search and analysis of the identification of scientific cognition stages, research methods and mechanisms for the implementation of intra-subject and interdisciplinary connections based on professionally oriented and research approaches; the attitude of the individual to self-determination and self-organization, the development of principles and styles of scientific thinking: induction, deduction, insight, analogy, inversion and anticipation:

1. *Mastering the standards and samples of visual modeling phenomenology* of a generalized construct and the results of diagnostic procedures (variability of definitions, ways of representation and conditions of existence – historiogenesis, practice – orientability, experimental and applied methods and procedures in the study of standards and samples of synergy manifestations; verification of analogies and associations of a generalized construct, computer and mathematical modeling of specific manifestations of generalized construct essence.

2. *Creation of a motivational field for choosing the content of research activities* and an actualization of personal preferences. Visual modeling (empirical tests and historiogenesis, multiplicity of goal setting, an actualization of mathematical knowledge and information technology, distribution of the roles in small groups and consideration of personal interests) of functional, operability and content of motivational and applied situations of standards and patterns in various interpretations of synergy (based on the choice of "problem zones" and complex knowledge generalized construct).

3. *Setting of multi-stage mathematical and informational tasks* to actualize the deployment of individual educational trajectories for school student's small groups (determining the composition and orientation of small groups, role allocation, selection and actualization of practice-oriented research activities according to the stages of founding and adaptation of generalized construct);

- *approximate – information saturation (self-determination ("what can I do"))* in the implementation of empirical tests and the design of visual models and the variability of founding procedures for representing particular of an essence manifestation for «problem zones" generalized construct in mathematical education based on the symbiosis of mathematical and computer modeling. The process implementation of significant connections identifying and continuity of empirical generalizations, awareness of the functionality of mathematical content level in an essence manifestation of generalized construct and correction in the state of its parameters and conditions, the adequacy and effectiveness in the correlation of "goal-result" orientation. The modes of basicity and integrativity of designed constructs are implemented as an indicative and informational basis for purposeful and variable research activities of school students (construction, calculation, variations, applied tasks, resource and laboratory calculation classes, computer algebra systems Maple, MathCad, MathLab using, an identification of patterns and presentations).

Designing the search content and research activities:

1. Components, actualization and organization of the adaptation processes of "problem zones" generalized construct to the content of school mathematics (contradictions and accessibility of mathematical apparatus and methods: graphs of knowledge and methods coordination, visual modeling and founding procedures; an actualization of synergy attributes and knowledge integration, search for stable clusters of empirical generalizations and applications (pedagogical support, laboratory and computational classes, resource classes, design methods, computer design and computational procedures, presentations, business games, scientific conferences and seminars);

2. Multiple goal-setting of research processes of "problem zones" generalized construct:

2.1. Identification of the content, stages of an essence founding of generalized construct, formalization, historiogenesis, presence of an essence manifestation samples at the reference and situational levels;

2.2. Multiple experience in solving of mathematical education micro-problems in "warming up" mode and the development of supra-situational activity (emotional experience, reflection, visual modeling, insight, solution verification, transfer); possibilities analysis of ICT tools to verify the adequacy of complex problems solving by using mathematical methods; an independent formulation of tasks and methods of their solution;

3. Readiness for a discussion and a multiplicity of problem solutions; an identification of criteria for the selection, formulation and search for solutions to research practice-oriented tasks based on diagnostic-information systematized in the form of founding complexes;

- *process-activity (self-organization ("I am able to control the process"))* is manifested in the design and technology organization for the development of research procedures by school students for the development of an essence innovative manifestations for generalized construct. It is realized during the deployment of its founding stages and on the basis of updating the techniques of creative cognitive self-activity and dialogue of mathematical, informational, natural science and humanitarian cultures. At the same time, forms, methods and means of mastering the generalized construct are being developed and implemented, adequate to their local, modular and global manifestations of the deployment of funding procedures:

1. Creating a creative environment in the process of an essence mastering of generalized construct (stimulating of success situation; working in small groups and cultures dialogue; tolerance to uncertainty; an identification and popularization of creative behavior samples and its results); collection and variety of forms and methods of information presentation, probabilistic-statistical, content, graphical, cluster, mathematical analysis development of statistical packages and office editors, small computer tools, computer algebra systems and Web support;

2. Development of divergent thinking and self-organization against the background of integrative constructs mastering, taking into account the probable and improbable circumstances, constructing the content, stages, basic and variable characteristics of an object design; theoretical and empirical generalization of knowledge and methods, an integration of knowledge and methods against the background of new quality interaction, actualization and formation in the "zones of immediate development" of personal experience;

3. The ability to adapt and develop in social communications based on the dialogue of mathematical, informational, natural science and humanitarian cultures in the context of computer and mathematical modeling of an adaptation components and stages of "zones of modern achievements in science" to school mathematics. Interaction, mutual influence, and mutual enrichment of knowledge fields are realized, which gives an idea of different ways of cognition and awareness of reality (rational natural science and irrational humanitarian) based on the openness of information environments, non-linearity of thinking (logical and intuitive), and is a factor in the development of post-non-classical values.

The *manifesting process of knowledge and procedures synergy* is implemented in the stages according to the actualization selected levels of cultures dialogue in the deploying direction of founding didactic procedures for equipping and mastering the generalized construct essence of "zone of modern achievements in science" and obtaining of probabilistically guaranteed results in teaching mathematics:

- *structural and logical level of knowledge and procedures integration* of various disciplines in the context of dialogue and unity of cultural diversity in the development of school students (in small groups, business games, network interactions, presentations, scientific conferences and seminars) synergy patterns at the reference and research situational level of specific natural science and humanitarian problems by using mathematical and computer methods;

- *level of actualization the unity and peculiarities of cultures dialogue* in the diversity of intercultural communication in productive development of an essence stages deployment of generalized construct. At the same time, the basis of cultures dialogue is an essence manifestation of generalized construct at this studying stage. Additionally, the following forms can be implemented: project activities, WebQuest, trainings;

- *level of self-organization and self-development of intercultural interactions* in the context of an essence actualization of generalized construct (the appearance of by-products, transformation of forms and methods, variation of parameters and conditions of cultural dialogue, the emergence of sustained interest and value attitude towards other cultures, development of integrative courses and programs with the goals and results of achieving synergetic effects);

4. *Visual modeling of integration* (coordination graphs) of mathematical, informational, humanitarian and natural science knowledge at the stages of an essence manifestation; creation of situations of intellectual tension and self-organization of school students, actualization of uncertainty and bifurcation points of mathematical procedures, mechanisms of self-determination and self-actualization in problematic situations during the development of generalized construct's an essence component;

- *control and correction stage* (evaluation of results empirical verification) – manifests itself in the design of monitoring functions and stages and diagnostic procedures for measuring the state and an experience expansion, development of mental functions, dynamics of synergetic effects and characteristics of school student's personal qualities:

1. Definition and optimization of technological procedures and an education subject content, mastering levels of the essence and deployment stages of founding spirals and clusters; definition of personal experience an integral complex of founding spirals and clusters in the course of generalized construct's an essence mastering as a necessary component of didactic field and the basis of processes variability at modern achievements in science adaptation;

2. Taking into account the probable and improbable circumstances, evaluating their effectiveness, the ability to set and solve problems in uncertainty conditions; self-analysis of solution's strategies and methods an effectiveness, choosing the optimal way to problem solving;

3. An assessment of the hypotheses, forecasts and strategies truth, their modification, evaluation of methods and procedures for finding results, variation of conditions and data of the tasks;

4. An actualization of synergy attributes (bifurcations, attractors, fluctuations, pools of attraction) in the process of generalized construct studying:

- *forms*: distance learning of project groups, laboratory and calculation classes, multi-stage mathematical and information classes, scientific conferences and seminars, networking and discussion forums;

- *tools*: mathematical and computer modeling, Qt Creator – a cross-platform free IDE for C++ development, pedagogical software products, WebQuest as a means of Web technologies with academic subjects integrating, Wiki sites, Messenger, Skype;

- *technologies*: coordination graphs of mathematical knowledge and procedures, work in small groups, WebQuest – as a technology of self-organization in collective creativity, project method, Wiki technology, visual modeling and founding of personal experience;

- *generalizing-transformative (self-development of personality ("I can do something new"))*. It is characterized by: the content and characteristics of an innovations transfer into the mass practice of school mathematics mastering; the integration of individual and social in the design of innovative generalizing constructs; information exchange, socialization and verification of innovative activities; characteristics, parameters and indicators of the formation and severity of school student's an individual educational trajectory.

An innovation content:

1. The supra-situational level of thinking, the desire to overcome stereotypes, harmonization of reflexive outputs, new creative products, an assessment and prediction of further actions, motivation for self-actualization;

2. An identification of patterns, analogies, associations, dynamics of studied processes, phenomena and facts;

3. The forecast and the "by-products" of the research (video clips, design methods, computer design and intelligent systems, web quests, presentations).

In this regard, special attention in synergy structure of technological stages functioning is paid to the consideration of the problems of school student's organizing group interaction, which is the most important source of their self-actualization and development, an incentive for creative activity and further personal growth. When organizing group creative the activity, it is necessary to create the conditions for generating a multiplicity of solutions to the problem based on information enrichment, intellectual tension and a low degree of behavior regulation. Thus, in the group form of research activity, school students have the opportunity to show the supra-situational activity and implement techniques for activating creative thinking in mutual dependence, an actualizing the dynamics of creative process: intuition, verbalization, visual modeling, formalization, reflection, verification, based on the synthesis of convergent and divergent thinking.

Example. The implementation of technological constructs for synergy manifestation and school student's creative potential growth in the course of "problem zone "surface area"" studying based on an area's growth dynamics of multifaceted surfaces at cylinder lateral surface ("Schwartz's boot").

Implementation of the stages:

- *Motivational field.* Visual modeling of motivationally applied situations on "new" interpretation of surface area concept (polyhedral surfaces area, an impossibility of sphere unfolding, rotation surface,

regular Schwartz cylinder (uniform division of the base into n sectors and height into m layers – Fig.2.), surface area of the sphere, cylinder, cone, torus according to the method of G. Minkowski.

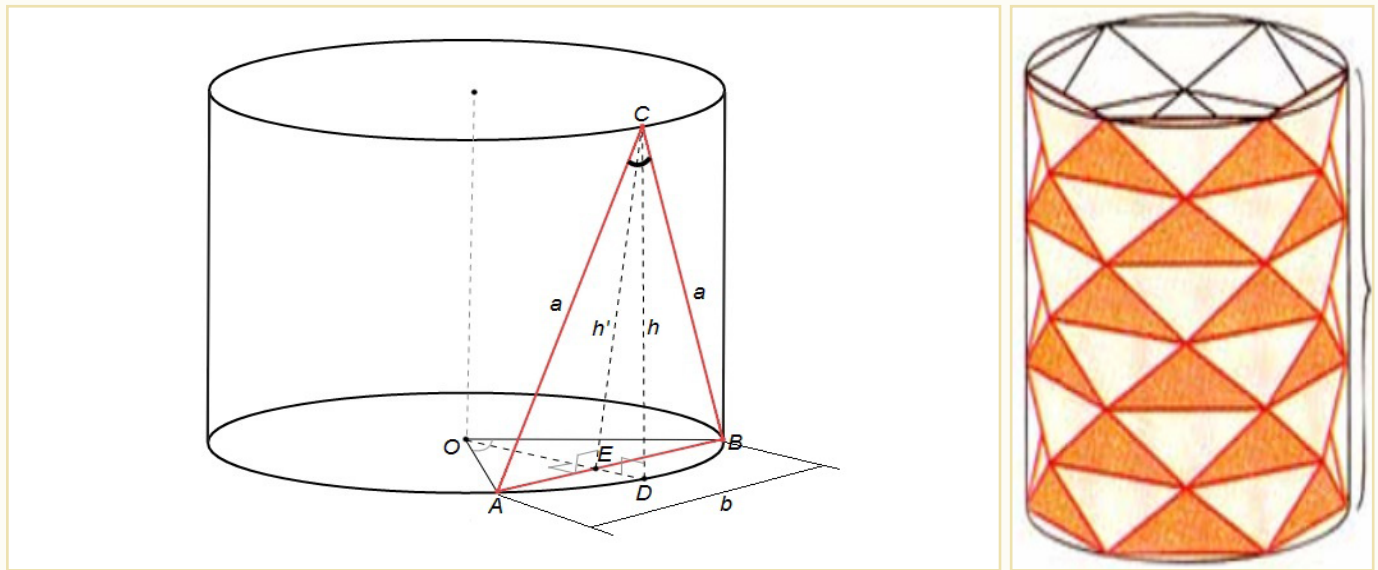


Figure 2 Triangulation element of surface layer side of Schwartz cylinder

Tasks for updating the "problem zones" for small groups of school students:

- computer and mathematical modeling of polyhedral surfaces an area finding (construction, calculation, properties, variations, laboratory calculation classes);
- the existence and an area surface finding of smooth curves rotation around the axis; sweep of the cylinder and cone, the impossibility of sphere sweep (computer and mathematical modeling, variations, applied tasks, resource activity);
- finding of Schwarz cylinder surface "area" in regular case ($m < n^2$, $m = n^2$, $m > n^2$) (computer and mathematical modeling, construction, calculation, variations, applied tasks, laboratory and computational classes);
- computer and mathematical modeling of surface areas finding of a sphere, cone, cylinder, torus according to the method of G. Minkovsky (construction, calculation, variations, applied problems, laboratory and calculation classes);
- computer and mathematical modeling of finding the polyhedral surfaces areas with an irregular height division and different rotation angles of layers.

• *Multiple goal-setting of research processes of scientific knowledge generalized construct* (an actualization of surface area concept by methods of Schwartz cylinder "area" studying (content aspect)): pathological properties of the of cylinder's lateral surface "area" have been well studied in the so-called "regular" case (see, for example, [23]). This happens when its height H is divided into m equal parts (respectively, the cylinder layers), and the circle lying at the base is divided into n equal parts, followed by a shift φ on each layer by π/n . With such triangulation of cylinder lateral surface, the formula for calculating its "area" by means of polyhedra resulting for $m, n \rightarrow \infty$ has the form:

$$S_q = 2\pi R(R^2 n^4 q^2/4 + H^2)^{0,5} \quad (1)$$

where:

$$q = \lim_{m,n \rightarrow \infty} (m/n^2) \quad (2)$$

Thus, the "area" of lateral surface S_q of a regular Schwarz cylinder of height H and radius R (if this limit exists – finite or infinite) is completely determined by the limit (2). It is clear that the true

value of lateral surface area ($q=0$) can be obtained by considering the tangent planes at triangulation points and then moving to areas limit of an external polyhedra with unlimited splitting of partitions. At the same time, due to the independent nature of the aspiration m , $n \rightarrow \infty$ the result of limiting process becomes poorly predictable, ambiguous with patterns absence in chaotic unfolding of polyhedra fractal structures. B. Mandelbrot [12] showed that when $m = n^k$ the area of polyhedral surface grows as m^k ($k \neq 2$).

It's easy to see that $\lim_{q \rightarrow \infty} S_q = \lim_{q \rightarrow \infty} cq$, where $c = R^2 n^3$. Thus, to calculate the "area" of the lateral surface S_q of a regular Schwarz cylinder at sufficiently large values of q instead of formula (1), you can use the formula:

$$S_q = R^2 n^3 q \quad (3)$$

In the work of E.I. Smirnov, V.V. Bogun and A.D. Uvarov [23], the behavior of function (1) and the angle between triangulation triangles with a common base if $m = f^n(a_0) \cdot n^2$ and $m, n \rightarrow \infty$, where $f(a_0) = xa_0(1 - a_0)$ is a logistic mapping adequate to the scenario of P. Ferhulst [15], is investigated.

The authors obtained the following bifurcation diagram (Fig. 3) using the information technology (Q_t Creator environment).

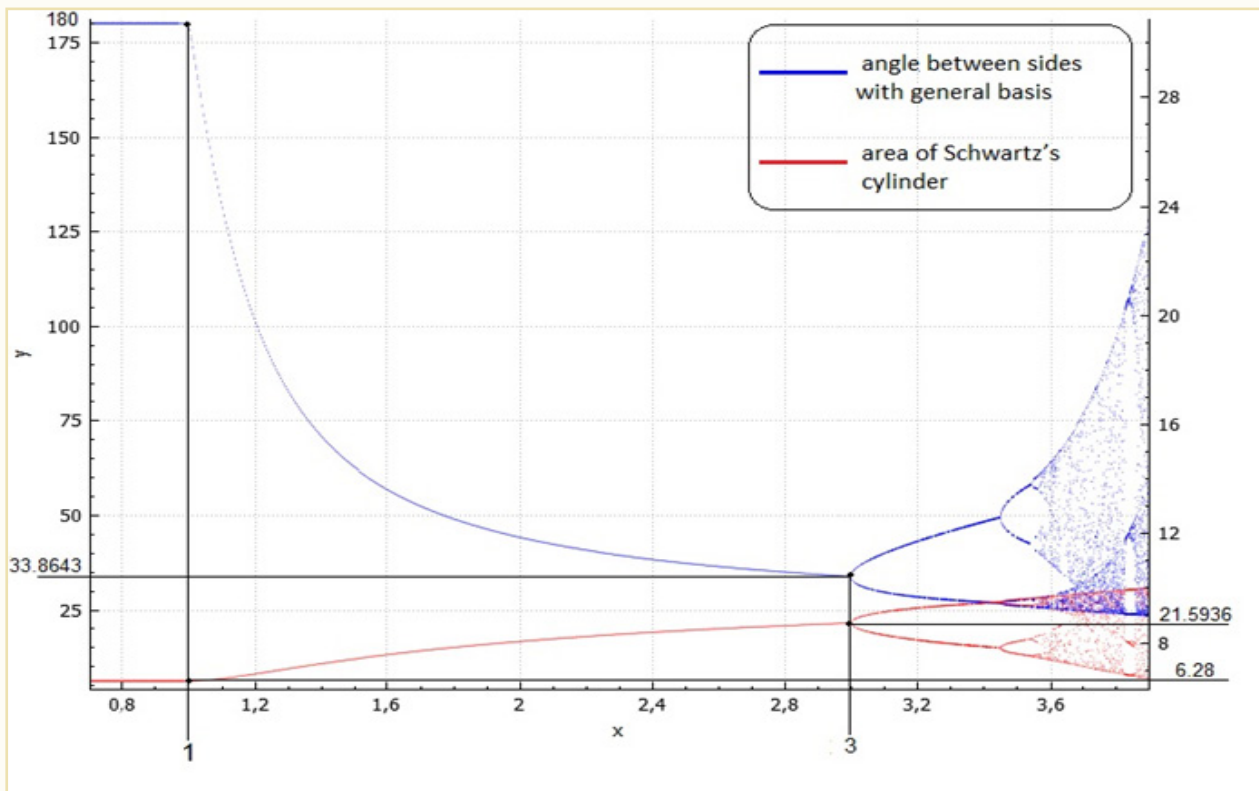


Figure 3 Bifurcation diagram of an areas and angles growth between the faces of triangles in a logistic mapping

Figure 3 shows two bifurcation diagrams at once. At the same time, the values of an angle between the faces with a common base, calculated by the formula, are postponed on the left vertical axis, and on the right axis, the values of the "area" of Schwartz cylinder calculated by formula (1) are postponed, taking into account $R=H=1$, at the same time, n varies from 500 to 1000.

Hierarchies of issues related to Fig. 3 arise and are solved by means of computer and mathematical modeling of school student's research activities in small groups in a remote environment or in the form of multi-stage mathematical and informational tasks studying with an irregular division of cylinder height.

• *Identification of by-products.* Consider a regular n -gon inscribed in a circle of radius 1. It is not difficult to determine the function $f_n(\alpha)$ and the following function $g(\alpha)$ as a functional series that determines the corresponding remainder of the radius to circle arc after crossing with the side of the n -gon. where the functions $f_{k \cdot n}(\alpha)$ are defined. The geometric meaning of the function $g(\alpha)$ is as follows: let's write all the right $k \cdot n$ – angles into the circle at the same time, where $k = \text{const}$, and $k \geq 3$. For each angle α (α – the rotation angle of radius), we will assign a numerical convergent series, each member of which is the length of the segment enclosed between the circle arc and the intersection point of radius with the side of n -gon. Next (Fig. 4) is an example of a partial sum $S_{50}(\alpha)$ of the series for $k=3$.

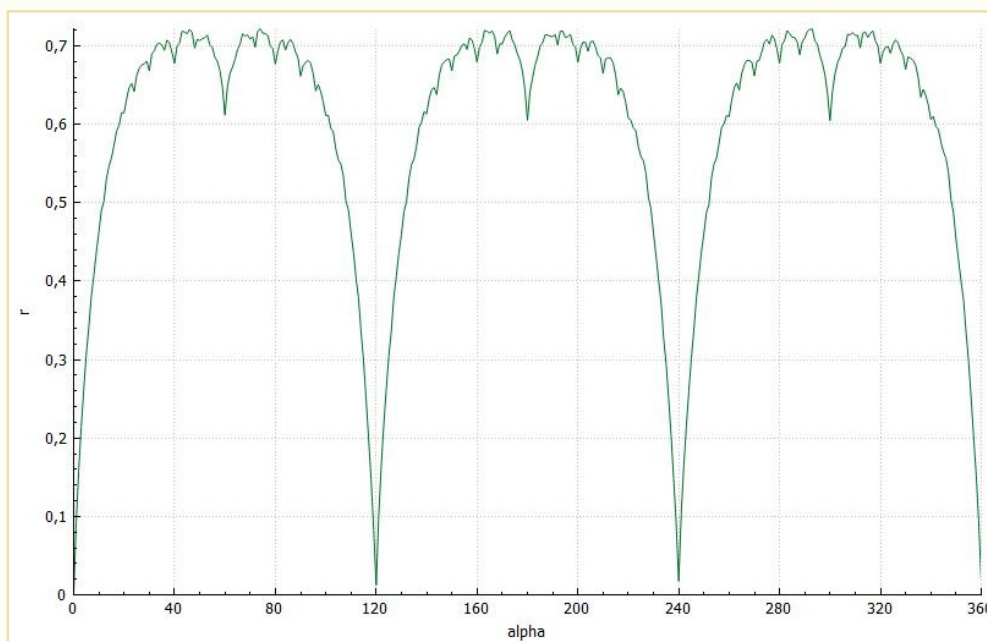


Figure 4 Fractal curves on the cylinder cross section

From Figure 4, it is easy to see that the graph of the function has a fractal structure, similar to the graph of Van Der Waerden function for a segment.

Now let's consider the layer of Schwartz cylinder intersected by an orthogonal plane to its axis (Fig. 5).

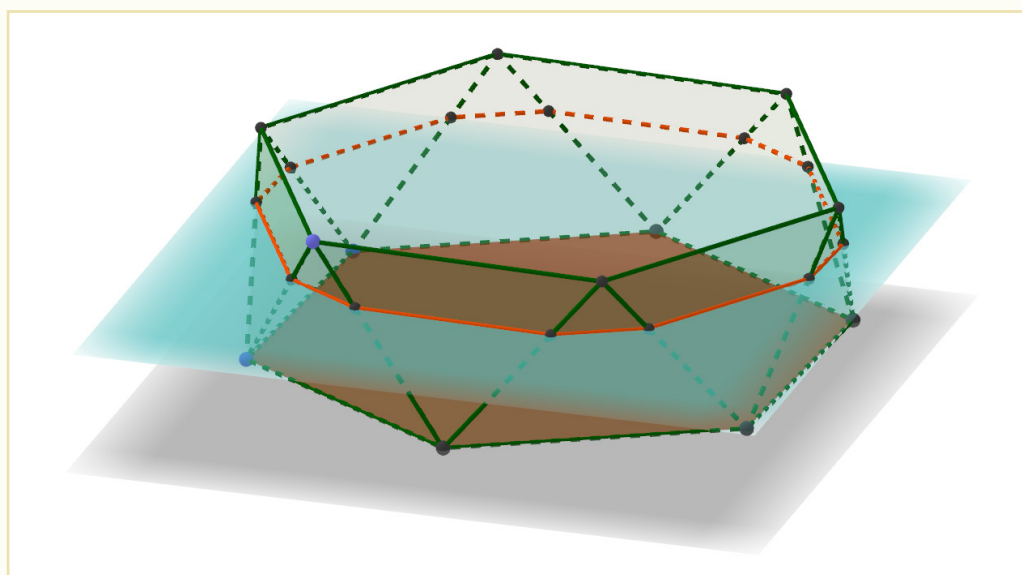


Figure 5 Cross section of Schwartz cylinder's polyhedral surface

A natural task arises: Let's give a Schwartz cylinder of height $H = 1$ and radius $R = 1$. At the same time, its upper base is divided into n equal parts, and its height into m equal parts. Let's draw a section perpendicular to the axis of cylinder through an arbitrary point x on it. If it tends to infinity, but is fixed, then what kind of function will be defined?

To solve this problem, we will assume that the point x moves within the first layer. Let's introduce a one-dimensional coordinate system that coincides with the axis of the cylinder. Beginning of the system is located in the center of the upper base, and the direction from the upper square.

Formula shows that the radius of circle described near the section, at a fixed n , is completely determined by the angle ε , which naturally changes from 0 to φ when the section moves from the upper base to lower one.

It can be shown that the angle ε is uniquely determined by the position of point x . Consider the following Figure 6.

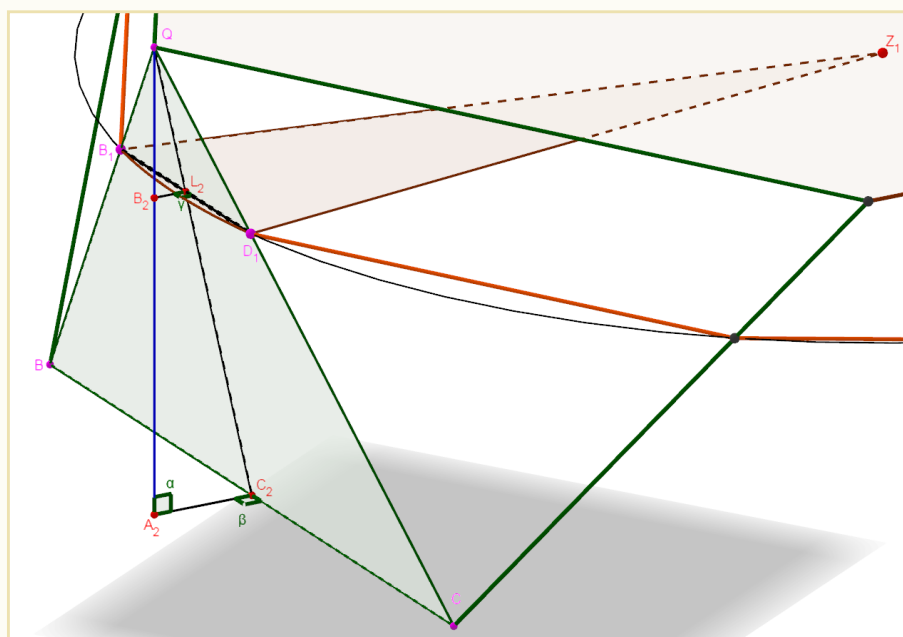


Figure 6 A fragment of the cross section of Schwartz cylinder

In Figure 6 we have $\varepsilon = \angle B_1Z_1D_1$. We will assume that for $n \rightarrow \infty$ the length of chord B_1D_1 tends to the length of circle arc B_1D_1 with the center at the point Z_1 and the radius $D_1Z_1 = h_3$, where h_3 is determined as $h_3 = (\sin Y / \sin \beta) = (\sin(90^\circ - 90^\circ/n)) / (\sin(90^\circ + 90^\circ/n - \varepsilon/2))$. Let be QA_2 is the height of the layer, B_2 – point of intersection of height QA_2 with the plane of section passing through the point x . This result is an approximate formula for finding the angle value through the ratio in which the point divides the height of layer.

It should be noted that formula allows you to find the angle ε value for a section passing through any point $x \in [0; 1]$.

Thus, each whole angle α uniquely defines an angle $s(\alpha \bmod \varphi)$.

Note that formula for $g(\alpha)$ can be expressed not only in terms of angle ε , but also in terms of the ratio of the height of the section to the height of the layer Earlier it was said that formula allows you to express the value of the angle ε in terms of point position x on cylinder axis. If we assume that the variables α and ε in formula for $g(\alpha)$ are independent, then the series defined by this formula can be viewed as a function of two variables $s(\alpha, x)$.

The following Figure 7 shows a part of the surface $z = s(\alpha, x)$, with $0^\circ \leq \alpha \leq 360^\circ$ and $0 \leq x \leq 1/30$.

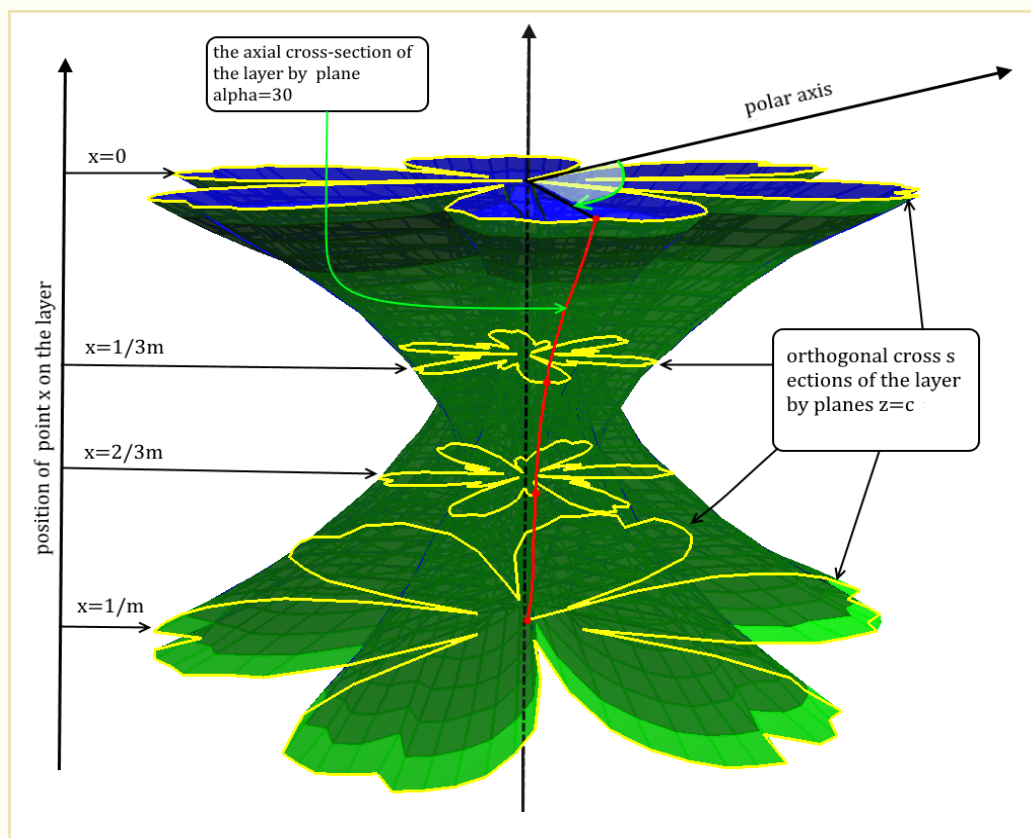


Figure 7 Fractal surface $z=s(\alpha,x)$, with $0^0 \leq \alpha \leq 360^0$ and $0 \leq x \leq 1/30$

We assume that the height of one layer of Schwartz cylinder is equal $1/30$ (for clarity, the surface is depicted in a cylindrical coordinate system). In the last figure, the level lines shown in yellow correspond to the graphs of function $g(\alpha)$ in the polar coordinate system at $k=0$, $k=1/3$, $k=2/3$, $k=1$. Figure 7 clearly shows how the orthogonal section lying on the upper base of Schwartz cylinder layer "rotates" when the secant plane moves along the axis of layer to its lower base. The coordinate line is shown in red $\alpha=300$. The resulting images suggest that the surface area $z=s(\alpha,x)$ is not determined by the limit $\lim_{m,n \rightarrow \infty} (m/n^2)$, where m is the number of base sides of Schwartz cylinder, and n is the layers number of Schwartz cylinder. Obviously $\lim_{n \rightarrow \infty} S_z = \infty$, where S_z is the surface area $z=s(\alpha,x)$.

It should be noted that the fractal curves, surfaces and bodies obtained in this study were obtained for the first time, but neither their volumes, areas and lengths, nor the fractal dimensions of the obtained by-products have been studied at present. At the same time, the variety of practice-oriented and generalized construct-related (surface area) tasks allows for variability and consideration of each school student's personal preferences. The synergy manifestation of complex knowledge based on the symbiosis of mathematical and computer modeling contributes to the formation of mathematical and digital literacy and each school student's creative potential growth. It is proved through the experimental testing of innovative technology by means of more than 30 developed and implemented elective courses for specialized high schools.

DISCUSSION

The problem of "avoiding complexity" and the tendency to simplify the processes of mathematical abstractions mastering, unfortunately, is practically resolved in modern mathematical (and not only) education and teacher training (A. Hašková [7] – context of teacher's competences evaluation, A.I. Kreyk [8] – synergy aspects of organic modernization, R.R. Maaliw III [9] – adaptive virtual learning

environment, N. Meseşan [13] – cultivation of functional literacy). These trends are reflected in their research by the E. Morin [14] – complexity in education, J. Santos [18] – innovative pedagogical practices, R.I. Semenova [21] – STEAM-education and IT-employment as factors of adaptation, V.K. Egorov [30] – human capital development, K. Mainzer [11] – complex dynamics of matter, mind, and mankind, convincingly demonstrates that a transition is necessary in the modern world from linear thinking to non-linear thinking, especially in the upbringing of the younger generation. E. Morin already at the beginning of the XXI century draws attention to the need for a radical reform of thinking: "The reform of thinking is a key anthropological and historical problem. Never in the history of mankind has the responsibility for thinking and culture been as crucial as it is today. Complex thinking is not a replacement of simplicity with complexity, but the implementation of a continuous dialogical movement between simple and complex" (E. Morin [14]). Therefore, the paradigm of complex systems and knowledge mastering becomes relevant as a factor of each school student's intellectual development against the background of an emotional response to the applied effect and the symbiosis of mathematical and computer modeling in teaching mathematics during the search and research activities. This article develops a technology and methodology for complex systems and knowledge mastering in teaching mathematics based on synergy manifestation in the study of generalized constructs of modern achievements in science. Thus, an alternative to the paradigm of "simplification" in mathematical education is presented with a proven effect of school student's an intellectual operations thinking and increasing creativity and motivation to study mathematics.

CONCLUSION

The technology of "problem zones" identifying and researching in teaching mathematics allows the integrating knowledge from various fields of science, creates a precedent for each school student's research activities, actualizes the synergetic effects in mastering process of complex knowledge and contributes to the of each school student's creative potential growth. It is the possibility of studying on the basis of multiple goal-setting the manifestations of bifurcation transitions and fluctuations in limiting processes of complex systems and knowledge generalized constructs, their adaptation to school mathematics, that creates the effect of increasing the creative potential of each school student. At the same time, the manifestation of synergetic effects is realized and an attractor and an attraction pool of iterative processes in various coordinate spaces are visualized on the basis of mathematical and computer modeling. The identification and study of "problem zones" in teaching mathematics by means of computer and mathematical modeling allows them to master of basic educational elements. It can be studied the modern achievements in science, such as: fuzzy sets and fuzzy logic, elements of fractal geometry, theory of information encoding and encryption, cellular automata, etc. At the same time, the openness of the educational environment, multiplicity of goal-setting and possibility of obtaining by-products create the basis for an effective development of intellectual thinking operations, increase the educational motivation, creativity and self-organization of individual in the context of intercultural communications.

REFERENCES

1. Abramova I.V., Vechtomov E.M., Shilova Z.V. Innovative aspects of the organization of design and research activities of teachers at the university. *Perspectives of Science and Education*, 2020, no. 3 (45), pp. 488-507. DOI: 10.32744/pse.2020.3.35. (in Russ.)
2. Dvoryatkina S.N., Smirnov E.I. Evaluation of synergetic effects of integration of knowledge and activity based on computer modeling. *Modern information technologies and IT education*, Moscow, MSU Publ., 2016, pp. 35-42. (in Russ.)
3. Dvoryatkina S.N., Melnikov R.V., Smirnov E.I. Technology of synergy manifestation in the research of solution's stability of differential equations system. *European Journal of Modern Education*, 2017, vol. 6, no. 4, pp. 684-699. DOI: 10.13187/ejced.2017.4.684.
4. Fatahi S. An experimental study of an adaptive e-learning environment based on the personality and emotions of the student. *Education and Information Technology*, 2019, vol. 24, no. 4. pp. 2225-2241. DOI: 10.1007/s10639-019-09868-5.

5. Rui Luo, Zhinan Peng, Jiangping Hu. On Model Identification Based Optimal Control and Its Applications to Multi-Agent Learning and Control. *Mathematics*, 2023, no.11(4), 906, DOI: 10.3390/math11040906.
6. Haken, H. Principles of brain functioning: A synergetic approach to brain activity. *Behavior and cognition*. Springer, 1996, 363 p.
7. Hašková A., Malá E. Training Needs of Technology Teachers in the Context of Teacher s Competences Evaluation. In A. Sandu, T. Ciulei, & A. Frunza (Eds.), Logos Universality Mentality Education Novelty. *European Proceedings of Social and Behavioural Sciences*, 2019, vol 15, pp. 464-473. DOI: 10.15405/epsbs.2016.09.60.
8. Kreyk A.I., Komf E.V. Synergy Aspects of Organic Modernization. In K. Anna Yurevna, A. Igor Borisovich, W. Martin de Jong & M. Nikita Vladimirovich (Eds.), Responsible Research and Innovation. *European Proceedings of Social and Behavioural Sciences*, 2019, vol. 26, pp. 444-450. DOI:10.15405/epsbs.2017.07.02.57.
9. Maaliw R.R. III. Adaptive virtual learning environment based on learning styles for personalizing e-learning system: design and implementation. *International Journal of Recent Technology and Engineering*, 2020, vol. 8, no. 6, pp. 3398-3406. DOI: 10.35940/ijrte.F8901.038620
10. Malinetsky G.G., Potapov A.B., Podlasov A.V. Nonlinear dynamics: approaches, results, hopes. Moscow, URSS Publ., 2006, 279 p. (in Russ.)
11. Mainzer K. Thinking in Complexity: The Complex Dynamics of Matter, Mind, and Mankind: 2nd and extended Edn. New York: Springer, 1996. Japanese translation: Tokyo, 1996.
12. Mandelbrot B.B. Fractal geometry of nature. Moscow, Institute of Computer Research, 2002. 656 p.
13. Meseşan N., Albulescu I. The Cultivation of Functional Literacy by Holistic Education Literature Review. In V. Chis, & I. Albulescu (Eds.), Education, Reflection, Development – ERD 2018, vol 63. *European Proceedings of Social and Behavioural Sciences*, 2019, pp. 192-202. DOI:10.15405/epsbs.2019.06.25.
14. Morin E. Method. Nature of Nature / E. Morin. Moscow, Progress–Tradition, 2005, 464 p.
15. Ostashkov V.N., Smirnov E.I., Belonogova E.A. Education Sinergy in Research of Attractors and Basins of Nonlinear Mapping Attraction. *Yaroslavl Pedagogical Bulletin*, 2016, no. 6, pp. 146-157. (in Russ.)
16. Poddiakov A.N. Psychology of teaching/learning under conditions of novelty, complexity and uncertainty. *Psychological Studies*, 2015, vol. 8, no. 40. DOI: 10.54359/ps.v8i40.558. (in Russ.)
17. Prigozhin I. Nonequilibrium statistical mechanics. Moscow, Mir Publ., 1964, 314 p. (in Russ.)
18. Santos J., Figueiredo A.S., Vieira M. Innovative Pedagogical Practices in Higher Education: An Integrative Literature Review. *Nurse Education Today*, 2019, vol. 72, pp.12-17.
19. Schwartz H.A. Sur une définition erronée de l'aire d'une surface courbe. *Gesammelte Mathematische Abhandlungen*, 1890, no. 1, pp. 309-311.
20. Sekovanov V.S. Elements of the theory of discrete dynamical systems. Saint-Petersburg, Lan' Publ., 2016, 180 p. (in Russ.)
21. Semenova R.I., Zemtsov S.P., Polyakova P.N. STEAM-education and IT-employment as factors of adaptation to the digital transformation of the economy in the regions of Russia. *Innovations*, 2019, no. 10 (252), pp. 58-70. DOI: 10.26310/2071-3010.2019.252.10.006. (in Russ.)
22. Smirnov E.I. Foundation of experience in professional training and innovative activity of a teacher. Yaroslavl, Chancellor Publ., 2012, 654 p. (in Russ.)
23. Smirnov E.I., Bogun V.V., Uvarov A.D. Synergy of mathematical education: An introduction to analysis. Yaroslavl, Chancellor Publ., 2016, 216 p. (in Russ.)
24. Tongxin Li, Weiping Wang, Xiaobo Li, Tao Wang, Xin Zhou and Meigen Huang. Embedding Uncertain Temporal Knowledge Graphs. *Mathematics*, 2023, no. 11(3), 775, DOI: 10.3390/math11030775.
25. Pei – Shan Hsieh. Will virtual reality increase children's hope, pleasure, and travel inspection. *Psychology and Behavioral Science International Journal*, 2021, no. 5(17), pp. 9-19, DOI: 10.10980/PBSIJ.2021.17.555974.
26. Dvoryatkina S.N., Shcherbatykh S.V., Smirnov E.I. Technological stages of Schwartz cylinder's computer and mathematics design using intelligent system. *Special Issue on Multidisciplinary Innovation in Engineering Science & Technology organized by Advances in Science, Technology and Engineering Systems Journal (ASTESJ Publishers)*, 2021, USA, no. 6(1), pp. 447-456. DOI: 10.25046/aj060148.
27. Hyunsuk Han. The utility of receiver operating characteristic curve in educational assessment. *Mathematics*, 2022, no. 10(9), 1493, DOI: 10.3390/math10091493.
28. Po Ling Chen, Tedd Denis. Cognitive and social-emotional skills training: why should we consider board games. *Psychology and Behavioral Science International Journal*, 2022, no. 4(18), pp. 19-28, DOI: 10.10980/PBSIJ.2022.18.555992.
29. Smirnov E.I., Uvarov A.D., Tikhomirov S.A. The manifestation of synergy in the study of multy-stage mathematical and information tasks based on the parameterization method. *CONTINUUM. Maths. Informatics. Education*, 2024, no. 1 (33), pp. 18-33. DOI: 10.24888/2500-1957-2024-1-18-33 (in Russ.)
30. Egorov V. K. Human capital and effective development. *Economic consultant*, 2020, vol. 30 (2), pp. 76-88. doi: 10.46224/ecoc.2020.2.7

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