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High school students’ computational thinking process to solve derivative calculus problems

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ABSTRACT

The problem and the aim of the study. Derivative problems can be found through a computational thinking process which includes the stages of decomposition, finding patterns, algorithms, abstraction, and generalization. However, high school students are used to using formulas for solving derivative problems and still find it difficult to present problems in visual form, symbols and words. This situation illustrates the problems of high school students today. *This study aims* to explore and elucidate the computational thinking processes of high school students as they tackle derivative problems.

Research methods. This study aims to find the process of high school students' computational thinking skills in solving derivative problems through descriptive analysis. Data were collected through problem-solving tests and interviews. The subjects of this study included high school students from three State High Schools in Klaten, Indonesia. The instruments consists of two problems which include the problem of finding higher order derivatives (problem 1) and solving for maximum value (problem 2). Interviews were conducted with two representative students (student 1 = SS and student 2 = BD).

Results. Students have solved the derivative calculus problem. Some students are able to solve the derivative calculus problem according to the stages of the computational thinking process. Although only 76.92% (problem 1) and 70.51% (problem 2) of students correctly articulated the information, these results are considered substantial. Pattern recognition remains relatively low, with only 20.51% (problem 1) and 19.23% (problem 2) of students accurately identifying derivative solving strategies. However, 28.57% (problem 1) and 32.05% (problem 2) of students exhibited algorithmic thinking. At the generalization stage, only 17.95% (problem 1) and 19.23% (problem 2) of students were able to do it. At the abstraction stage, only 64.10% (problem 1) and 51.28% (problem 2) of students were able to do it.

Conclusion. The results of the study showed that there was a computational thinking process in solving derivative problems. The decomposition process can be seen in students writing information on the problem using symbols, writing keywords, and writing the requested problem by including a question mark (?). The pattern recognition process can be seen from the steps in solving the given problem, such as several experiments and systematic stages. The algorithmic thinking process of problem solving can be seen from the explanation of each step of the solution: writing formulas, finding derivatives of the function f(x), finding the nth derivative, getting the volume in the form of V(x), finding the first derivative equal to zero, finding x, finding the size of the box, and finding the maximum volume. The generalization process can be seen in writing the results of calculating derivatives and maximum volume. The abstraction process can be seen in answering the problem.

KEYWORDS

computational thinking, derivative calculus, thinking process, problem- solving

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INTRODUCTION

UNESCO has made computational thinking an important skill for the 21st century, even as a complement to the 4C abilities (critical thinking and problem solving, creativity, communication, and collaboration), which are really needed by future generations [1]. Computational thinking skills are recognized as a crucial aspect of modern education, significant not only for those pursuing interests in computer science and mathematics but also essential for every student in the twenty-first century. The development of computational thinking hones various abilities, including problem-solving, analytical, critical, creative, and innovative thinking, as highlighted in several key studies [2]. They [3] think that computational thinking processes are very useful for solving mathematical problems in elementary and high school levels. In addition to elementary school students and high school students, research [4] states that computational thinking skills can foster critical thinking skills for students in college.

Fundamentally, it involves a cognitive process where in students comprehend the context of a problem, advance through stages of abstraction, and ultimately arrive at systematic solutions, enabling them to tackle complex problems methodically ([5; 6]). Research by [5] and [6] further emphasizes that students with strong computational thinking skills can engage in sequential and organized problem-solving, enhancing academic performance and real-world preparedness. The integration of computational thinking into curricula improves proficiency across subjects, encourages interdisciplinary learning, and equips students with versatile skills applicable in diverse fields. As educators and policymakers increasingly recognize its importance, incorporating computational thinking into educational frameworks becomes essential for preparing students to thrive in a digital and data-driven world.

Although computational thinking skills are critically important, Indonesian students have yet to achieve satisfactory results in this area. The 2018 PISA report by the Organization for Economic Co-operation and Development (OECD) revealed that the average mathematics score for Indonesian students was only 379 [7]. PISA assessments are divided into six proficiency levels, with levels four, five, and six encompassing indicators such as problem identification, reflective thinking, problem formulation, interpretation, evaluation, generalization, and the application of information to mathematical problems. These indicators align with the components of computational thinking skills. The 2018 PISA report also indicated that fewer than 10% of Indonesian students achieved proficiency at levels four, five, or six in mathematics. Complementing these findings, a study by [8] assessed high school students' computational thinking skills, focusing on algorithmic thinking, decomposition, abstraction, evaluation, and generalization. The study found that over 46% of students scored below 50, placing them in the low computational thinking skills category [8]. Furthermore, fostering computational thinking skills remains a significant challenge for teachers within mathematics education. These findings highlight the urgent need for targeted interventions and instructional strategies to enhance computational thinking among Indonesian students, ensuring they are better prepared for the demands of the twenty-first century.

Developing computational thinking skills is an essential learning outcome in mathematics education, particularly in topics such as derivative calculus. In the current Indonesian high school curriculum, derivative calculus is studied in the twelfth grade, within phase F. One of the key learning outcomes for derivative calculus is the ability to solve real-world problems involving derivatives, such as determining maximum and minimum values [9]. Solving derivative problems involves a process that includes translating real-world contexts into mathematical form, solving the mathematical problem, and then interpreting the solution back into the original context [10]. This approach not only enhances students' understanding of mathematical concepts but also improves their problem-solving skills by linking theoretical knowledge to practical applications. Additionally, fostering computational thinking through derivative calculus can prepare students for higher education and various professional fields where analytical and critical thinking are crucial. Therefore, it is imperative that educators emphasize these skills in their teaching practices to ensure that students are well-equipped to tackle complex problems in both academic and real-world settings.

The study of derivative calculus is crucial for developing students' computational thinking skills; however, many high school students still struggle with solving derivative problems. Fatmanissa et al. [11] identified various difficulties encountered by high school students when solving derivative problems involving maximum area, maximum height, and maximum volume in word problems. These difficulties include challenges in transforming problem-solving processes into visual, symbolic, and linguistic forms. Their research highlighted that the most common difficulty faced by students was converting the language of derivative problems into visual representations, a critical step that simplifies the problem-solving process [11]. For instance, students struggled to visualize a problem involving the volume of an open-top box made from a piece of cardboard. Some students incorrectly visualized the cardboard as a square instead of a rectangle, and others had trouble visualizing the rectangular cuts at each corner, which should have been squares.

Another study provides quantitative data on the errors made by high school students in solving derivative problems. It was found that more than 50% of students failed to correctly express derivative concepts or derivative calculus theorems, over 60% made errors in applying basic techniques for determining derivatives, and more than 30% made mistakes in presenting problem solutions [12]. Previous research has identified various difficulties and errors students encounter when solving derivative problems; however, there has been a lack of studies that explain the computational thinking processes students use in tackling derivative problems. In study [13], the computational thinking abilities of middle school students in relation to number sequences were explored, while study [14] described the computational thinking skills in the context of algebra. These studies highlight the need for a more focused examination of how students employ computational thinking specifically when addressing derivative problems.

Given the importance of computational thinking skills, the learning outcomes of derivative calculus, and the current weak understanding of derivative calculus among high school students, research into the computational thinking processes of students in solving derivative problems is crucial. This study aims to explore and elucidate the computational thinking processes of high school students as they tackle derivative problems. Understanding these processes can provide valuable insights into how students approach problem-solving in mathematics, identify specific areas where they struggle, and develop targeted interventions to improve their computational thinking skills. By investigating how students translate real-world problems into mathematical form, solve these problems, and interpret the solutions back into their original contexts, this research seeks to enhance instructional strategies and support the development of stronger computational thinking abilities in high school mathematics education.

METHODS

This study is a descriptive research project aimed at investigating the computational thinking processes of high school students in solving derivative problems. Data were collected through tests and interviews to capture students' computational thinking processes. The test, which comprised derivative problems, was administered to 78 students across three high schools in Klaten, Indonesia: 27 students of State Senior High School 1 Klaten, 25 students of Karangdowo 1 State senior high School, and 26 students of state senior high school 1 Jatinom.

Computational Thinking Process Indicators

Nordby et al. [15] specifically elucidate computational thinking within the real of mathematics. Computational thinking is defined as encompassing skills such as sequencing, iteration, conditionality, debugging, decomposition, and abstraction, alongside process-oriented activities like communication, creativity, exploration, and engagement [15]. These learning activities are instrumental in solving mathematical problems. Research indicators [16] has written that computational thinking involves the process of thinking and computing. What are the aspects of computational thinking? According to researchers indicators [17], there are five aspects in the computational thinking process, including decomposing a problem, algorithmic thinking, generalizing patterns, detecting solutions, and evaluating solutions. Researcher [18] have presented computational thinking consisting of the process of decomposition, finding patterns, abstraction, and algorithms. Researchers [19] have written that aspects of computational thinking include thinking algorithms, decomposition, generalization, abstraction, and evaluation.

Indicators of computational thinking processes in solving derivative problems in this research consist of : decomposition, pattern recognition, abstraction, algorithms, and generalization. Table 1 provides an explanation of the computational thinking process, including indicators and sub-indicators used to assess students' computational thinking abilities in solving derivative problems.

Table 1

Description of the Computational Thinking Process

Computational Thinking Process	Indicators	Description
Information decomposition	a. Writing down information from the derivative problem. b. Listing and organizing information from the derivative problem.	a. Students can identify information within a derivative problem and recognize emerging issues. b. Students are capable of simplifying derivative problems. c. Students can transform derivative problems from verbal or written forms into mathematical symbols. d. Students can establish formulas or equations.
Pattern recognition	a. Pattern recognition. b. Presenting patterns in solving derivative problem.	a. Students are able to recognize patterns, features, and characteristics of given derivative problems. b. Students can identify patterns, which are then implemented in constructing solutions to derivative problems.
Algorithmic thinking	a. Describing patterns. b. Algorithmic thinking.	a. Students are able to describe patterns that will form solutions to derivative problems. b. Students can outline systematic and logical steps that can be used to develop solutions to derivative problems.
Generalization	Making decisions	The students generalize the solutions to derivative problem-solving.
Abstraction	Drawing conclusions	Students can formulate conclusions by eliminating unnecessary elements when implementing the derivative problem-solving plan.

Computational Thinking Test Instruments

The derivative problem worksheets served as the primary instruments in this research. These derivative problems were validated by high school mathematics teachers and university mathematics lecturers to ensure their appropriateness and accuracy. The derivative problems used to measure computational thinking processes in solving derivatives problems can be observed in the high-order derivatives problem (Figure 1) and the maximum volume problem (Figure 2).

Amanda and Rafa are in a study group team. They are asked to solve a high-level derivative problem, namely finding $f^{(21)}(x), f^{(60)}(x), f^{(90)}(x)$, dan $f^{(103)}(x)$ from the function $f(x) = x\cos x,\ n \in \mathbf{N}$.. They also discuss solving the problem with other group members.

(a) Amanda believes that solving the problem is done by finding $f^{(1)}(x), f^{(2)}(x), f^{(3)}(x),\ f^{(4)}(x)$, and so on until the requested derivative.

(b) Rafa believes that what Amanda did is not effective because it takes a long time. He also believes that it is better to just look for the pattern.

In your opinion, is the method used by Amanda effective in solving the problem?

If you find a more effective method like Rafa, explain your method and what are the results?

Figure 1 Instrument of Computational Thinking Process through Problem of Finding Higher-Order Derivatives (Problem 1)

Andi and Anton were given the task of making a box from a piece of cardboard. They only had a piece of cardboard. A piece of cardboard measuring 24 cm (length) and 15 cm (width). The cardboard will be made into a box without a lid by cutting the four corners into a square shape.

(a) Andi plans to make a box by cutting the four corners to measure 1 cm.

(b) Anton plans to make a box by cutting the four corners to measure 2 cm.

Will Andi or Anton get the box with the maximum box volume? Can you help them get a box with the maximum volume? Predict the volume of the box.

Figure 2 Computational Thinking Process Instrument through the Problem of Finding Maximum Volume (Problem 2)

Data Analysis of the Computational Thinking Process

The results of the derivative problem-solving test were analyzed according to computational thinking processes by calculating the percentage for each indicator. This approach quantifies the effectiveness of students in accurately solving problems relative to the total number of participants, thereby expressing their performance in percentage terms (%). This methodological framework provides a clear evaluation of how computational thinking strategies contribute to problem-solving capabilities among students, offering insights into their proficiency and areas for potential improvement in educational settings.

From the results of this derivative problem test, solutions categorized as highly proficient were selected and subsequently analyzed based on indicators of computational thinking ability. To bolster the descriptive analysis, unstructured interviews were conducted to validate the analyzed solutions. Two students, SS and BD, were selected as interview subjects to provide deeper insights into their problem-solving approaches and thought processes. These interviews aimed to provide qualitative data that complemented the quantitative analysis of the derivative problem solutions.

RESEARCH RESULTS

The description of the derivative problem-solving outcomes (Problem 1 and Problem 2) illustrates how the computational thinking process is depicted in Table 2.

Table 2

Description of Computational Thinking Process

Computational Thinking Process	Problem 1	Problem 2
Information decomposition	60 out of 78 76.92%	55 out of 78 70.51%
Pattern recognition	16 out of 78 20.51%	15 out of 78 19.23%
Algorithmic thinking	20 out of 78 28.57%	25 out of 78 32.05%
Generalization	14 out of 78 17.95%	15 out of 78 19.23%
Abstraction	50 out of 78 64.10%	40 out of 78 51.28%

When compared to other thinking processes, a significant majority of students have demonstrated the ability to articulate information from both Problem 1 and Problem 2. Although only 76.92% (problem 1) and 70.51% (problem 2) of students correctly articulated the information, these results are considered substantial. Pattern recognition remains relatively low, with only 20.51% (problem 1) and 19.23% (problem 2) of students accurately identifying derivative solving strategies. However, 28.57% (problem 1) and 32.05% (problem 2) of students exhibited algorithmic thinking. The percentage of students employing algorithmic thinking exceeds those utilizing pattern recognition. These findings suggest that problem-solving processes have improved compared to problem-solving strategies, indicating that students have a greater understanding of concepts such as derivatives, derivative properties, higher-order derivatives, and maximum values over various problem-solving strategies. Nonetheless, these percentages still fall short of 50%.

In solving the problem, student BD identified and reformulated key information from the given problem statement. The function $f(x) = x.\cos x$ served as the central keyword. BD explained that the problem was presented in a lengthy narrative format, prompting them to condense the essential details into a few key words for clarity and focus.

-Menurut saya cara yang digunakan Amanda kurang efektif karena memerlukan waktu yang cukup lama.
· Ya seharusnya mencari polannya terlebih dahulu agar lebi cepat pengerjaannya

I think the method Amanda used was not effective because it took quite a long time.
Yes, Amanda should have looked for the pattern first so that the solution could be faster.

Figure 3 The Abstraction Stage for Solving Derivative Problems 1

The solution approach adopted by student BD can be summarized into three steps: (1) calculating the first, second, third, and fourth derivatives, (2) identifying patterns in the nth derivatives, and (3) determining specific derivatives such as 21st derivative ($f(21)(x)$), 60th derivative ($f(60)(x)$), 90th derivative ($f(90)$), and 123rd derivative ($f(123)(x)$). These steps illustrate BD's systematic approach to tackling derivative problems, demonstrating a methodical process of derivative calculation and pattern recognition to achieve the desired outcomes in derivative calculus tasks.

From these steps, the student documented their solution sequentially. In the first step, they computed first derivative ($f(1)(x)$), by initially setting $u = x$ and $v = \cos x$, and then $u = 1$ and $v' = \sin x$. Using this information, the student determined $f(1)(x) = u'.v + v'.u = \cos x - x.\sin x$. Similarly, they proceeded to find second derivative $f(2)(x) = -2\sin x - x.\cos x$, third derivative $f(3)(x) = -3.\cos x + x.\sin x$, and fourth derivative $f(4)(x) = 4.\sin x + x.\cos x$. This systematic approach demonstrates the student's methodical process of deriving higher-order derivatives, employing differentiation rules and algebraic manipulation to accurately compute each subsequent derivative in the series.

SS explained that to find higher-order derivatives, they first calculated the first, second, third, and fourth derivatives, stopping after the fourth derivative. The fifth, sixth, seventh, and eighth derivatives followed a similar pattern, differing only in the leading numbers. Specifically, they pointed out coefficients such as 1 and 5, -2 and -6, -3 and -7, and 4 and 8 in these derivatives. This methodical approach allowed SS to systematically build upon their understanding of derivative calculus, focusing on the patterns and adjustments required to derive successive derivatives accurately.

In the second step, the student generalized the nth derivative by writing nth derivative ($f^{(n)}(x)$) for $n = 1, 5, 9, \ldots$, $n = 2, 6, 10, \ldots$, $n = 3, 7, 11, \ldots$, and $n = 4, 8, 12, \ldots$. Students write the nth derivative using the following four patterns.

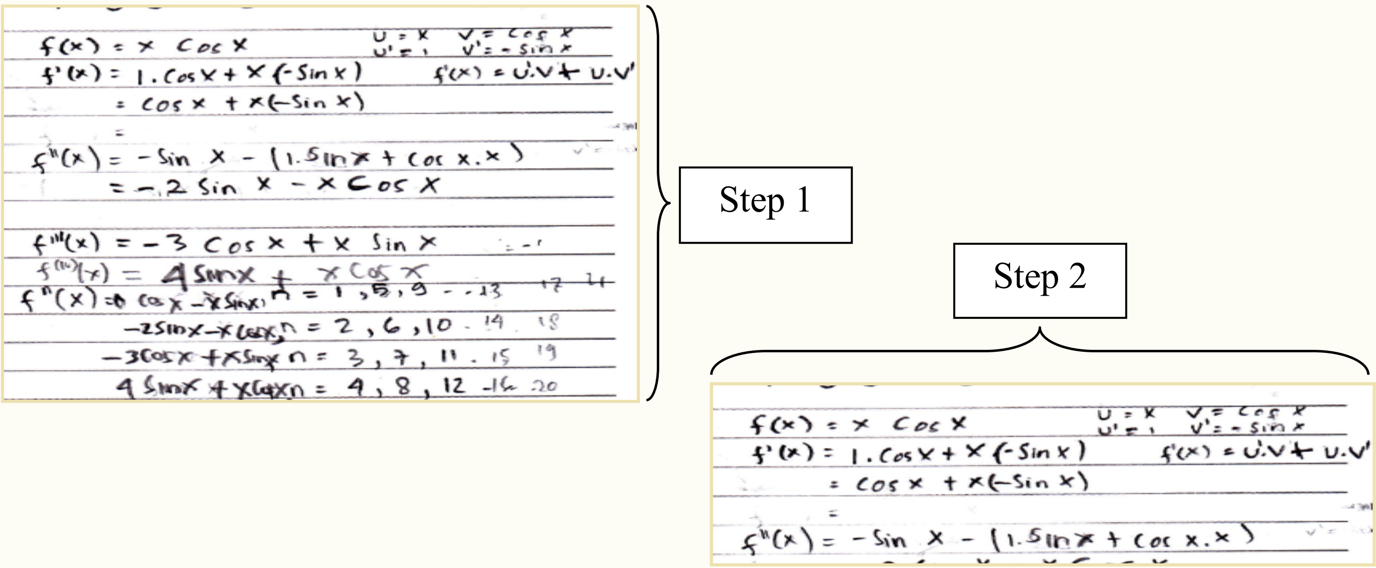

Figure 4 The Finding Patterns through Steps 1 and 2 for Derivative Problems 1

The student explained, "The pattern of the nth derivative is obtained by observing the results of derivatives from the previous steps," pointing out corresponding patterns. In the third step, the student identified the specific derivatives found : f(21)(x), f(60)(x), f(90)(x), and f(123)(x). This process involved applying the established patterns and calculations from earlier steps to determine these particular derivatives in derivative calculus problem-solving.

$$f^{(n)}(x)=\begin{cases} n.\cos x - x.\sin x,\ n=1,5,9,13,.... \\ \text{-}n.\sin x - x.\cos x,\ n=2,6,10,14,.... \\ \text{-}n.\cos x + x.\sin x,\ n=3,7,11,15,.... \\ n.\sin x + x.\cos x,\ n=4,8,12,16,... \end{cases}$$

The student elaborated on their findings, noting that the 21st derivative corresponded to the first pattern, the 60th derivative to the first pattern, the 90th derivative to the second pattern, and the 123rd derivative to the third pattern. BD successfully documented their answers as $f^{(21)}(x)$ = 21cos x − x.sin x, $f^{(60)}(x)$ = 60.sin x + x.cos x, $f^{(90)}(x)$ = -90.sin x − x.cos x, and $f^{(123)}(x)$ = −123. cos x + x.sin x in response to the high-order derivative question. These responses represent an abstraction stage. BD generalized the given problem effectively, indicating a structured approach to solving derivative problems. They also expressed dissatisfaction with Amanda's proposed method due to its time-consuming nature, opting instead for a more efficient strategy of identifying patterns beforehand to expedite the process. BD orally conveyed that this systematic approach could be applied universally to derive higher-order derivatives, reinforcing the notion that once a regular pattern is identified, the nth derivative for any n can be determined efficiently.

In solving the problem, student SS began by detailing the dimensions of the cardboard and its problem statement. SS expressed the dimensions of the cardboard using symbols: length of the cardboard, p = 15 cm, and width of the cardboard, l = 15 cm. SS articulated the problem as finding the maximum volume of a box, phrasing it with interrogative words and symbols (?). These steps indicate SS's ability to break down the information provided in the problem. SS mentioned that after reading the problem, they focused only on essential details and articulated the problem to be solved succinctly.

Student SS presented the problem-solving pattern effectively. The problem-solving pattern was carried out through three experiments: experiment 1, experiment 2, and experiment 3. In experiment 1, SS sketched a rectangle and listed the dimensions of the resulting cardboard, then determined the volume of the box formed by multiplying the three dimensions. In experiment 2, SS repeated the process from experiment 1. In experiment 3, SS sketched a rectangle with dimensions: length (p) = 24 cm, width (l) = 15 cm, and height (h) = x cm. These three experiments demonstrate that SS was able to present the problem-solving pattern effectively and accurately, showcasing their proficiency in understanding and applying geometric concepts in solving practical problems.

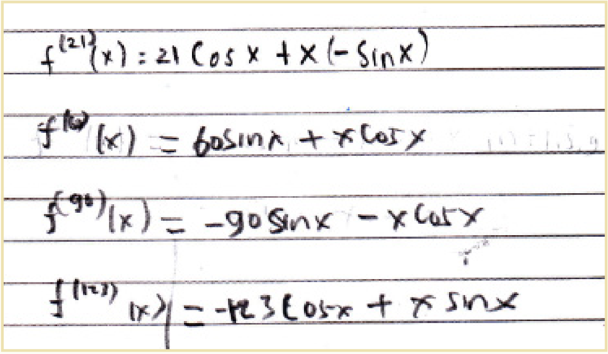

Figure 5 The Finding Patterns through Steps 3 for Derivative Problems 1

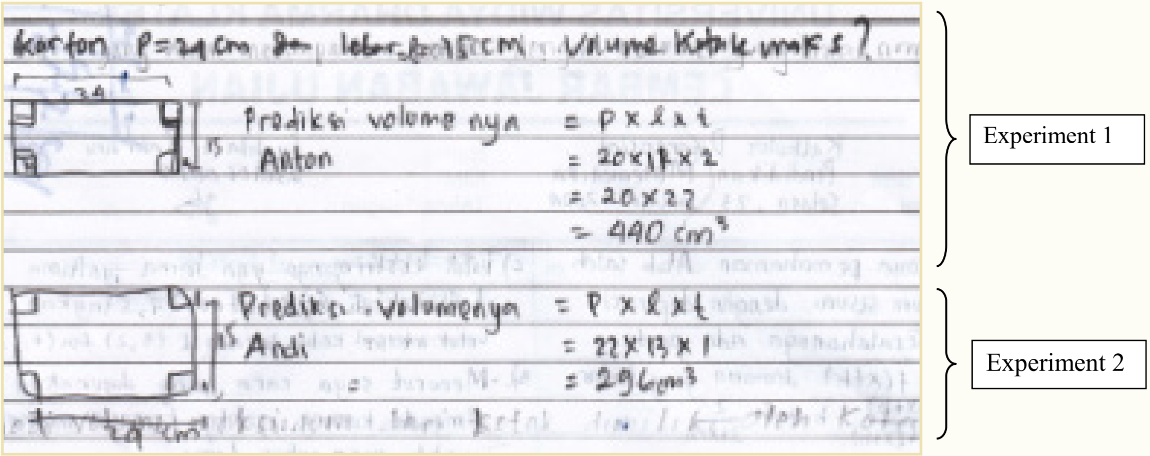

Figure 6 The Finding Patterns through experiments 1 and 2 for Problems 2

Based on the problem-solving approach presented, student SS effectively addressed the problem of finding the maximum volume of an open-top box from a piece of cardboard measuring length = 24 cm and width = 15 cm. In experiment 1, SS correctly determined the dimensions of the open-top box as p = 20, l = 12, and h = 2, resulting in a box volume of 440 cm³. SS accurately used symbols for length, width, and volume, and correctly applied the volume formula V = p × l × h with precise calculation and units. In experiment 2, SS also correctly identified the dimensions of the open-top box as p = 22, l = 13, and h = 1, resulting in a box volume of 296 cm³. SS consistently employed correct symbols for length, width, and volume, accurately calculating the box volume using the formula V = p × l × h with precise units and results. These experiments demonstrate SS's proficiency in applying mathematical concepts to solve practical problems effectively and accurately.

In experiment 3, student SS accurately depicted a rectangle sketch with correct dimensions : length = (24 – 2x), width = (15 – 2x), and height = x. Although not explicitly defining the variable x, SS clarified that x represents the height of the box to be formed. SS proceeded to calculate the variable volume of the box V(x) by multiplying the length, width, and height, resulting in V(x) = 4x3 – 78x2 + 360x. SS applied derivative concepts to find the maximum box volume, computing the first derivative V′(x) = 12x2 – 156x + 360 and setting it equal to zero. SS simplified the derivative by dividing each term by 12, yielding the simplest form V′(x) = x2 – 13x + 30 = 0. Further simplification led to factoring V′(x) = (x – 10)(x – 3) = 0, giving x = 10 or x = 3. Since substituting x = 10 resulted in a negative width (15 – 2x), SS concluded x = 3 was valid. Substituting x = 3, SS determined the dimensions: length = 18, width = 9, height = 3, and calculated the maximum box volume as 486 cm³. Compared to experiment 1 and 2, the volume achieved in experiment 3 was maximum, utilizing the correct application of the first derivative concept. SS successfully employed a methodical approach to solving the maximum volume problem, including sketching the rectangle, determining its dimensions, calculating the volume function V(x), using the first derivative to find x, and finally computing the maximum volume attainable.

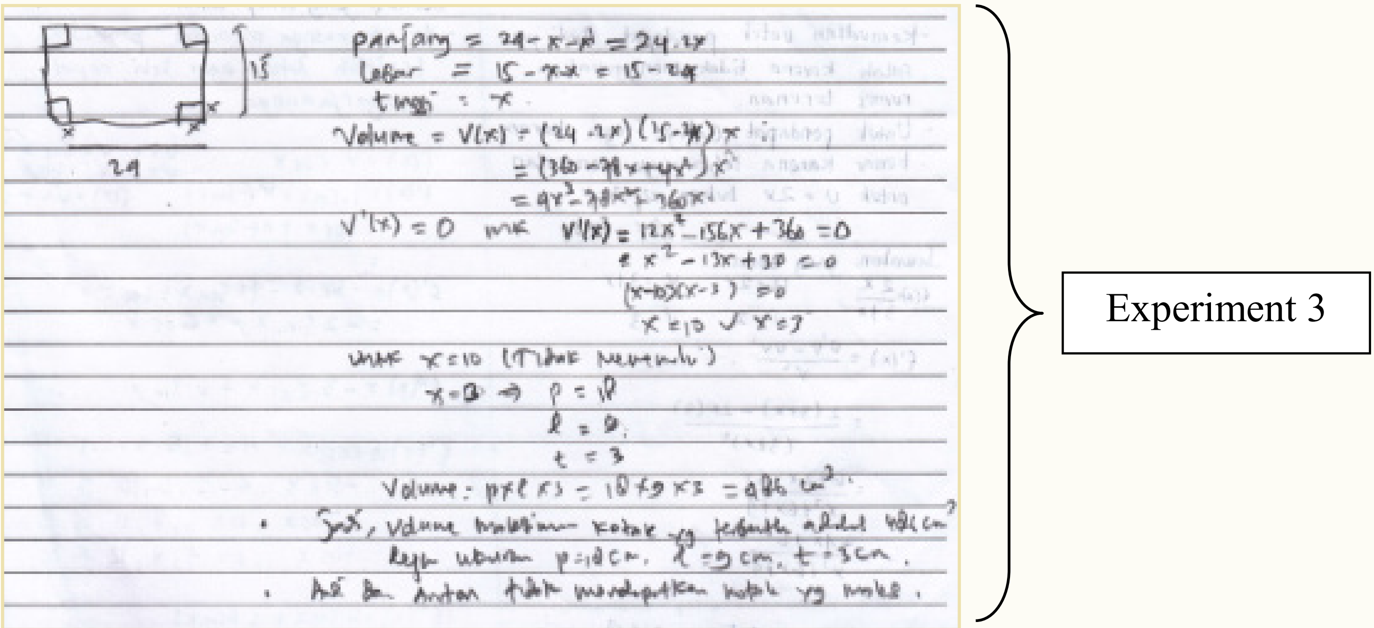

Figure 7 The Finding Patterns through Experiments 3 for Derivative Problems 2

Student SS has accurately written the abstract that V(x = 3) represents the maximum volume of the open-top box that will be formed, namely the maximum volume of the box being 486 cm^3. SS also generalized the problem-solving, stating "thus, the maximum volume of the box formed is 486 cm^3, with dimensions of length = 18 cm, width = 9 cm, and height = 3 cm." Additionally, SS addressed the problem's questions in their solution, noting that "the attempts by Andi and Anton would not achieve the maximum box volume," suggesting a step could be taken by cutting the corners of the rectangular sheet by 3 cm.

DISCUSSION

The problem-solving flow in finding high-level derivatives based on the computational thinking process is illustrated in Figure 8.

The problem-solving flow in finding the size of a block and the maximum volume of a block based on the computational thinking process is illustrated in Figure 9.

The computational thinking process in solving derivative problems encompasses five stages, as depicted in Figures 4 and 5. Mathematical problems can take the form of inquiries, as Lenchner [20] suggests. The derivative problems in this study are open-ended, characterized by soliciting students' opinions, seeking peer assistance to find solutions, predicting outcomes, and experimenting to discover answers. Addressing open-ended problems demands varying levels of creativity or originality [21] and [22]. According [22] that solving high-level mathematics through the process of coding, encoding, then solving the contextual problem. Solving these derivative problems requires precise steps to ascertain higher-order derivatives and determine maximum volumes. Prior to finding solutions, students need to comprehend the problem through multiple readings, then identify and note down keywords or relevant information, including the problem statement itself. Keywords can range from symbols, variables, formulas, functions, to entire sentences. In this process, students distill the narrative of derivative problems into simpler forms of information represented as keywords. Research by [23] underscores the critical importance of identifying keywords in narrative problems before devising problem-solving strategies.

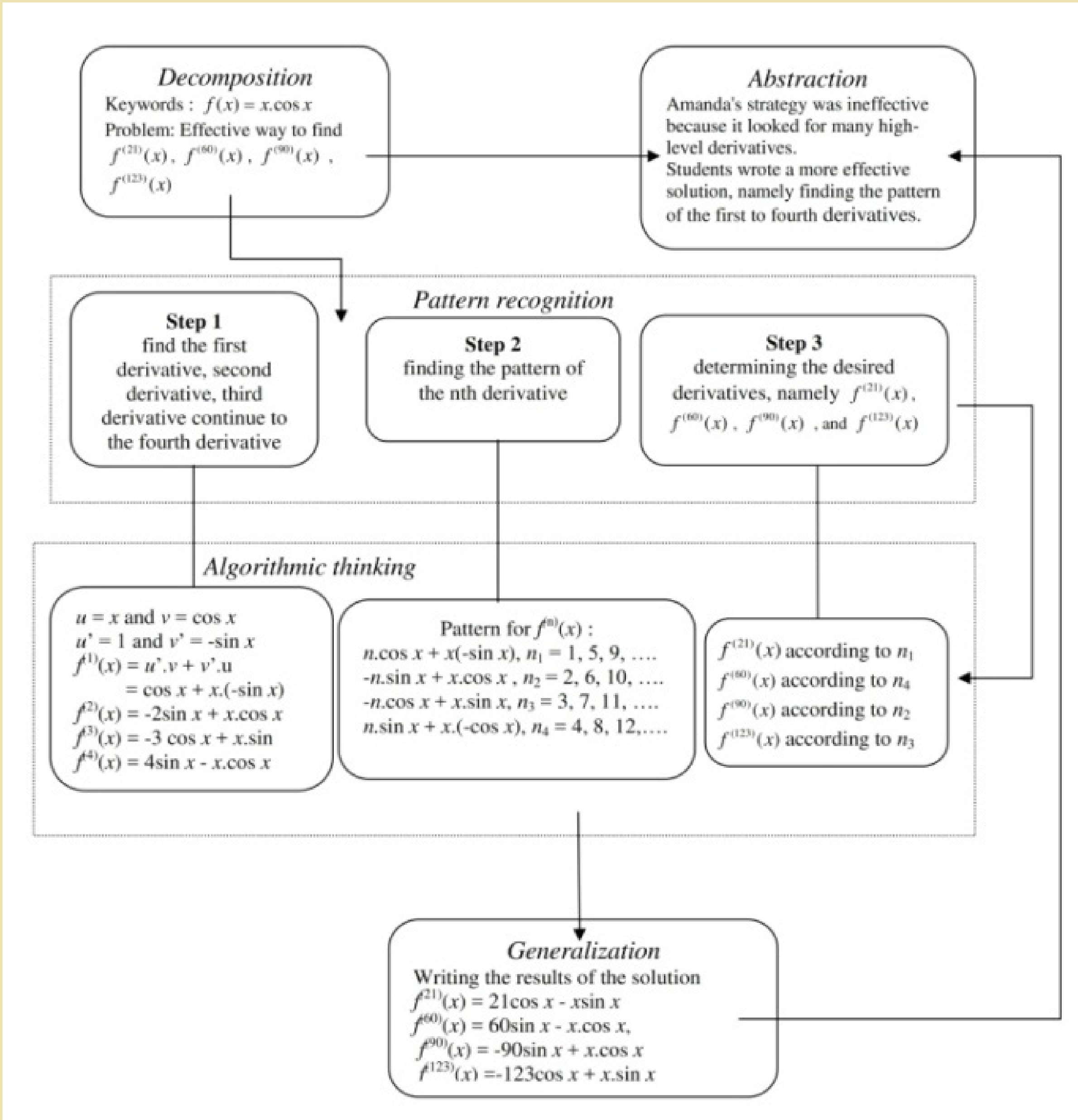

Figure 8 Computational Thinking Processes in Solving High Order Derivatives

To solve problems, students employ various strategies, particularly when dealing with open-ended problems. These strategies often require sequential steps and critical thinking to uncover solutions. In the context of solving derivative problems, students have utilized strategies such as experimentation, pattern recognition, and sketching. These problem-solving strategies have been applied in mathematical contexts [24], proving instrumental in visualizing solutions [25]. Research [26] has proposed solving mathematical problems through diagrams, listing, finding regular patterns, and so on, while [27] proving solves mathematical problems through inductive proof. Furthermore, students articulate their problem-solving approaches using concepts like derivatives, derivative properties, extreme value concepts, and the formula for calculating volume of a box.

In the final stage of computational thinking, students generalize and abstract the solutions they have derived from derivative problem-solving. They are able to restate their findings and articulate solutions either in sentences or through symbolic representations. As demonstrated by [28], generalize the solution of derivative problems using symbols. Based on this explanation, solving strategies has become an important aspect of computational thinking [29]. Various methods can be used to foster high school students' computational thinking in mathematics learning. Computational thinking can be enhanced through multimedia-based [30] and problem-based learning [31].

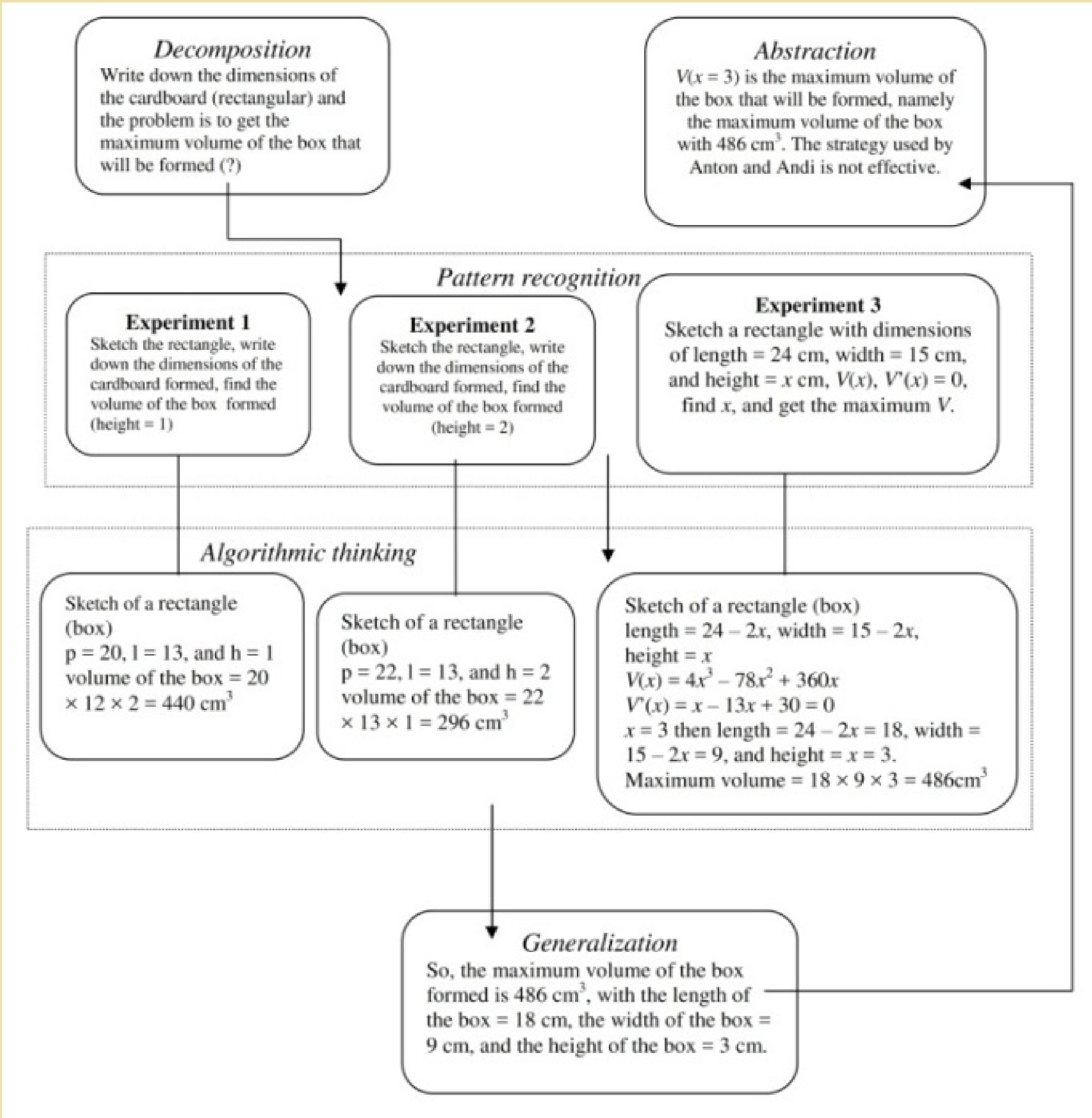

Figure 9 The Computational Thinking Process in Finding the Maximum Volume

CONCLUSION

The conclusion of this study shows that the computational thinking skills of high school students are categorized as low, medium, and high. The computational thinking process of students in solving derivative calculus includes decomposition, pattern recognition, algorithmic thinking, abstraction, and generalization. (a) The problem-solving decomposition process can be seen in students writing information on the problem using symbols, writing keywords, and writing the requested problem by including a question mark (?). (b) The process of recognizing problem-solving patterns can be seen from the steps in solving the given problem, such as several experiments (in special cases) until finding a general pattern. (c) The algorithmic thinking process of solving problems can be seen from explaining each step of the solution, writing formulas, finding derivatives of the function f(x), finding the nth derivative, getting the volume in the form of V(x), finding the first derivative equal to zero, finding x, finding the size of the box, and finding the maximum volume. (d) The generalization process can be seen from the results of calculating derivatives and maximum volume. (e) The abstraction process can be seen in answering problems.

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