Utilizing digital teaching aid to support functional thinking process of maritime vocational college students in solving linear interpolation problems based on differences mathematical ability

L. TARIDA, M. T. BUDIARTO, A. LUKITO

ABSTRACT

Introduction. The problem and the aim of the study. Vocational maritime students need to understand basic mathematical concepts to support shipping safety, for example, in the material on linear interpolation functions that can be used to estimate the relationship between various variables needed to avoid shipping accidents. However, students need help solving mathematical problems. Students require a functional thinking process to solve mathematical problems, primarily in generalizing the relationship between quantities. Some of them have difficulty in the functional thinking process. In this case, we utilize a digital teaching aid in the form of GeoGebra software, which has the potential to overcome the difficulties of functional thinking processes. Therefore, *this study aims* to explore the functional thinking process of maritime vocational college students in solving linear interpolation problems with GeoGebra to estimate relationships between variables that can avoid ship accidents.

Study participants and methods. This study involved (36) third-semester vocational maritime college students majoring in the Deck Officer Program in Indonesia with different categories of mathematical ability, including (11) high, (24) moderate and (1) low. This category was obtained based on their applied mathematics course scores in the first semester. A qualitative research approach was used in this study. Data collection was carried out by giving students problem-solving tasks (PST) and PST-based interviews. Data analysis began by calculating the frequency of occurrence of students' functional thinking aspects based on the students' PST answers, transcribing the interview results based on PST and organizing PST results and transcripts according to a framework that reflects the functional thinking process.

Results. The results showed that 50% of students achieved low functional thinking aspects with the frequency of occurrence of three out of six functional thinking aspects, 33.33% of students achieved four functional thinking aspects, and 16.67% of students achieved five functional thinking aspects. The functional thinking process of students with different mathematical abilities improved after they used GeoGebra to solve PST. Students with high and moderate mathematical abilities achieved all functional thinking aspects so that they discovered the general rules of linear interpolation used to estimate the relationship between variables to avoid ship accidents. Students with low mathematical ability also experienced increased functional thinking even though they only achieved four functional thinking aspects. The problem-solving stage can trigger functional thinking, including identifying problems at the analysis stage, representing data at the design stage, finding recursive patterns and finding covariational relationships at the exploration stage, finding correspondence relationships at the implementation stage, and evaluating general rules at the verification stage.

Conclusion. This study concluded that GeoGebra improved maritime vocational college students' functional thinking processes in problem-solving, especially in visualizing relationships through function graphs. However, students with different mathematical abilities showed different levels of engagement, and further research is needed to explore other types of tasks to improve functional thinking in all students.

KEYWORDS

functional thinking, problem-solving, digital teaching aid, GeoGebra, maritime

For Citation: Tarida, L., Budiarto, M. T., & Lukito, A. (2025). Utilizing digital teaching aid to support functional thinking process of maritime vocational college students in solving linear interpolation problems based on differences mathematical ability. *Perspektivy nauki i obrazovania = Perspectives of Science and Education*, (3), 560–572. https://doi.org/10.32744/pse.2025.3.37

Received: 12 December 2024 | Approved: 14 February 2025 | Published: 30 June 2025

INTRODUCTION

In the Mathematics Framework of the Programme for International Student Assessment (PISA) 2022, functional thinking is a cognitive process required in mathematical reasoning because of an important role played in understanding mathematical structures as students move up to higher grades [1]. Functional thinking is the main gateway to algebraic thinking, where algebra is central to mathematics and the gateway to higher-level mathematical concepts [2; 3]. The importance of functional thinking is marked by the development of related research since 1905 at the Meran Conference until now [4; 5]. Smith characterized functional thinking as representational thinking, highlighting the relationships between two or more varying quantities, specifically the kinds of thinking that lead from specific relationships (individual incidences) to generalizations of that relationship across instances [6]. Generalizing the relationship between varying quantities is essential for school mathematics and applying real-life contexts [7; 8]. In real-life maritime fields, functional thinking can be used to avoid accident risks because it makes quick and precise decisions by analyzing the relationship between accident-causing factors such as wind speed, ocean currents, and weather conditions to determine safe shipping routes [9]. The National Transportation Safety Committee (KNKT) found that the cause of the ship capsizing was inaccuracy in the calculation of ship stability related to considerations regarding the cargo capacity [10]. The accurate generalization of the relationship between quantities, such as the relationship between ship stability and cargo capacity, can reduce the risk of ship accidents. The importance of generalizing the relationship between quantities proves the crucial role of functional thinking for maritime students. However, research exploring functional thinking in maritime students still needs to be completed.

Students' functional thinking can be explored through functional thinking aspects. Table 1 presents the conceptualization of functional thinking aspects from several experts.

Table 1

The summary of experts' notions on functional thinking

Smith [6]	Blanton & Kaput [11]	Stephen et al. [12]	Pitta-Pantazi et al. [13]
• Recursive patterning • Covariational thinking • Correspondence relationship	• Pattern building • Conjecturing • Generalizing • Justifying mathematical relationships	• Relationships between quantities (recursive covariation and correspondence) • Multiple representations (pictures, words, variables, tables, and graphs)	• Find functional relations • Represent data • Predict unknown terms using known data • Identify patterns

Relationships between quantities consist of recursive patterning, covariational, and correspondence [12]. Recursive patterning involves finding variation within a sequence of values, covariational thinking is based on analyzing how two quantities vary simultaneously and keeping that change as an explicit, dynamic part of a function's description, and a correspondence relationship is based on identifying a correlation between variables [6]. Recursive patterning is defined as pattern-building [11] which is in line with identifying patterns [13]. Multiple representations [12] are used as a tool to find a functional relationship [13], which involves conjecturing and generalizing [11]. Justifying mathematical relationships [11] is needed in predicting unknown terms using known data [13]. Based on the literature review, we developed the functional thinking aspects used in this study (Table 2).

The emergence of students' functional thinking aspects can be triggered through problem-solving [14; 15]. Problem-solving is essential for students because it is a 21st-century skill needed to provide strategies to solve real-world problems across disciplines, such as the relationship between maritime science and mathematics [1]. This follows the body of knowledge from the International Association of Maritime Universities (IAMU), which states that problem-solving is one of the academic skills maritime students need in studying mathematics [16]. Problem-solving can be presented through tasks with non-routine mathematical problem criteria [17; 18]. Non-routine mathematical problems cannot be solved with routine procedures, so they require functional thinking processes. As previously mentioned, problems in the maritime context involve the interconnectedness of relationships to estimate the influence of several factors to avoid ship accidents [9; 19]. Linear interpolation estimates an unknown value between two known values connected by a straight line [20; 21]. Therefore, the task used in this study triggers students to find the concept of linear interpolation function in the context of tanker load capacity in maintaining ship stability.

Table 2

Coding scheme of functional thinking aspects

Code	Functional Thinking Aspects
FT1	*Identifying the problem* Noticing the relationship between two quantities
FT2	*Representing data* Transforming data from tabular to graphical form
FT3	*Finding recursive patterns* Finding changes in each variable value based on the graph
FT4	*Finding covariational relationships* Finding the relationship between two variables based on a graph to predict an unknown value based on a graph
FT5	*Finding correspondence relationships* Creating general rules based on the graph
FT6	*Evaluating generalization rule* Testing the correctness of the general rule

Schoenfeld's problem-solving stages were used in this study because they involve a cyclic process that encourages the emergence of students' functional thinking aspects [22; 23]. The problem-solving stages include analysis, design, exploration, implementation, and verification. The exploration stage is the primary step in generalizing the relationship between quantities [18]. The generalization process is obtained by changing the situation from concrete to abstract. Based on the results of a preliminary study, it was found that maritime students needed help abstracting functional relationships from real-world concrete situations. GeoGebra is believed to help overcome these obstacles because it makes it easier for students to visualize representations of relations between quantities [24; 25]. GeoGebra can be accessed online at https://www.geogebra.org/classic. Graphical representations in GeoGebra facilitate the initial conceptualization of how changes in one quantity affect simultaneous changes in other quantities [18]. Therefore, this study allows students to use GeoGebra to complete problem-solving tasks involving graphical representations.

In addition to allowing students to use GeoGebra to support successful problem-solving, this study also involves mathematical ability. Mathematical ability is one of the success factors in the problem-solving process [26]. Therefore, we would like to study how digital teaching aids (GeoGebra) support the functional thinking process of maritime vocational college students in solving linear interpolation problems based on differences in mathematical ability.

RESEARCH METHODS

This study was qualitative research to explore how digital teaching aid (GeoGebra) supports functional thinking processes of maritime vocational college students in solving linear interpolation problems based on differences in mathematical ability. This study involved 36 third-semester vocational maritime college students in Indonesia majoring in the Deck Officer Program with different categories in mathematical ability. Students' mathematical ability categories are obtained based on their scores in applied mathematics courses in the first semester. Mathematical abilities are grouped into three categories, namely, high mathematical abilities if the score (x) obtained is in the range ($80<x\leq100$), moderate mathematical abilities if the score (x) obtained is in the range ($60<x\leq80$), low mathematical ability if the score (x) obtained is in the range ($0\leq x\leq60$). After obtaining their scores, we had 11 participants with high mathematics abilities, 24 with moderate mathematics abilities, and 1 with low mathematics abilities.

Data was collected by asking participants to solve problem-solving tasks (PST) and then interviewing them based on their results. PST was given to participants in two different periods to explore the differences in functional thinking processes when using GeoGebra to help them succeed in problem-solving. In the first period, 36 participants worked on PST on a worksheet without the help of GeoGebra. The results of the first-period PST answers were intended to design categories of functional thinking emergence (see Table 3).

Table 3

Functional Thinking Categories

Functional Thinking Category	Frequency of Occurrence of Functional Thinking Aspects
A	Six aspects: FT1-FT6
B	Five aspects: FT1-FT5
C	Four aspects: FT1-FT4
D	Three aspects: FT1-FT3
E	Two aspects: FT1-FT2
F	One aspect: FT1

In the second period, three participants were selected to represent each category of mathematical ability, including one participant with high mathematical ability, one participant with moderate mathematical ability, and one participant with low mathematical ability. To prevent bias in the research findings, we selected participants with the widest range of scores in each category of mathematical ability. In addition, the selection of the three participants was also based on the highest frequency of functional thinking aspects in each category of mathematical ability. So, each selected participant had the highest functional thinking aspect representing their mathematical ability category. The three participants were then labelled HMA for participants with high mathematical ability, MMA for participants with moderate mathematical ability, and LMA for participants with low mathematical ability. In this second period, the three participants were given laptops to access GeoGebra, which was used as a tool for working on the PST. They worked on the PSTs one by one at different times so that the researchers could focus more on making observations. This difference also provides an opportunity to explore the use of GeoGebra in more depth through interviews based on the PST results.

PST is designed based on the functional thinking aspects in Table 2. PST contains non-routine problems related to linear interpolation. Linear interpolation is chosen because it can be used as a solution to shipping safety problems. The application of linear interpolation to PST helps estimate the relationship between the height and load capacity of a tanker ship. The estimated results of this relationship can prevent overloading that causes the ship to sink. PST has been validated by three experts who stated that the instrument is suitable for research with an average percentage of 84.22%. Improvements to PST, according to expert suggestions, have also been made. PST translated from Indonesian to English is presented in Figure 1.

The table below provides simulation data on cargo space height (m) and cargo space capacity (DWT) on a Tanker Ship

Cargo space height ("m")	3	6	9	12	15	18	21
Cargo space capacity ("DWT")*	5	10	12	17	19	23	25

**cargo space capacity in* 1×10^4 *DWT*

1. If the cargo space height is 14"m" , what is the estimated capacity?
2. What general rule do you use in estimating the cargo space capacity of the tanker? Please explain it!

Figure 1 PST in English

Functional thinking process begins when participants identify a problem by noticing the relationship between two quantities: cargo space height and cargo space capacity (FT1). Based on this relationship, participants will be more detailed in noticing which quantity influences (independent variable) and which quantity is influenced (dependent variable). The data on the relationship between the two quantities presented through a table does not show an implicit pattern, so it can trigger participants to represent the data in a graphical form (FT2). Graphical representation facilitates the initial conceptualization of how changes in one quantity affect changes in other quantities [18]. Graphical representation makes it easier for participants to find recursive patterns representing changes in each variable (FT3). In addition to graphical representation, finding the relationship between cargo space height and cargo space capacity is needed to predict an unknown value, namely cargo space capacity, when the cargo space height is 14"m" (FT4). Thus, a solution can be obtained for question 1. To find the general rule in question 2, participants will be involved in finding correspondence relationships (FT5). The sentence "Explain it" in question 2 triggers participants to test the correctness of the general rule (FT6).

The PST results data and interview transcripts were analyzed using the functional thinking aspect coding scheme in Table 2, which is linked to Schoenfeld's problem-solving stages, including analysis, design, exploration, implementation, and verification [22].

RESEARCH RESULTS

The results are presented based on the research aims to explore how digital teaching aids (GeoGebra) support the functional thinking process of maritime vocational college students in solving linear interpolation problems based on differences in mathematical ability. To achieve this aim, we grouped the categories of participants' functional thinking aspects based on the PST results of 36 participants. These categories are associated with different mathematical abilities (see Table 4). Explanations of each category from A to F can be seen again in Table 3.

Table 4

Distribution of functional thinking categories by mathematical ability

Mathematical Ability	Category of Functional Thinking						Sub-Total of Students
	A	B	C	D	E	F	
High	0	4	7	0	0	0	11
Moderate	0	2	5	17	0	0	24
Low	0	0	0	1	0	0	1
Total of Students	0	6	12	18	0	0	36

Based on the grouping of functional thinking aspect emergence categories, 6 participants (16,67%) were obtained with category B; 12 participants (33,33%) with category C; 18 participants (50%) with category D. Based on the grouping of mathematical abilities, there were 11 participants (30,56%) with high mathematical abilities, 24 participants (66,67%) with moderate mathematical abilities and 1 participant (2,78%)with low mathematical abilities. There were 4 participants in Category B and 7 in Category C of the 11 participants. There were 2 participants in Category B, 5 in Category C and 17 in Category D of the 24 participants. Finally, 1 participant with low mathematical abilities was included in category D. Grouping of students based on mathematical abilities and functional thinking categories is also presented in graphical form (see Figure 2). Most participants were in category D and had moderate mathematical abilities.

Utilizing GeoGebra to support functional thinking was explored more deeply through task-based interviews. The functional thinking process is presented in three parts based on the differences in participants' mathematical ability and the highest functional thinking aspect category, including high (HMA) in Category B, medium (MMA) in Category B, and low (LMA) in category D. Furthermore, it was analyzed based on six aspects of functional thinking (FT1, FT2, FT3, FT4, FT5 and FT6).

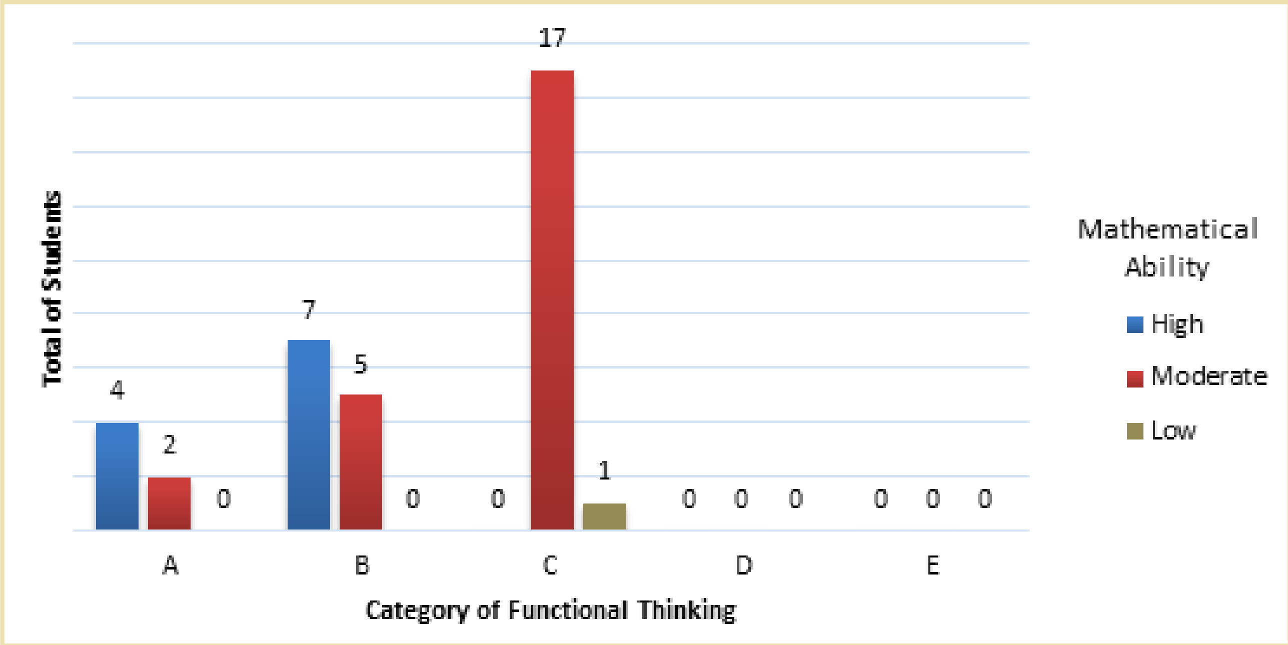


Figure 2 Number of participants based on functional thinking and mathematical ability categories

1. The student with high mathematical ability (HMA)

FT1: Identifying the problem

Based on the PST answers and interview transcripts, HMA began to understand the problem of tanker loading capacity after reading the PST in its entirety. HMA identified a relationship between height and tanker loading capacity (FT1). The relationship was further analyzed through interviews, which informed that the higher the height, the higher the tanker loading capacity. According to HMA, the height of the cargo hold affects the tanker's loading capacity.

FT2: Representing data

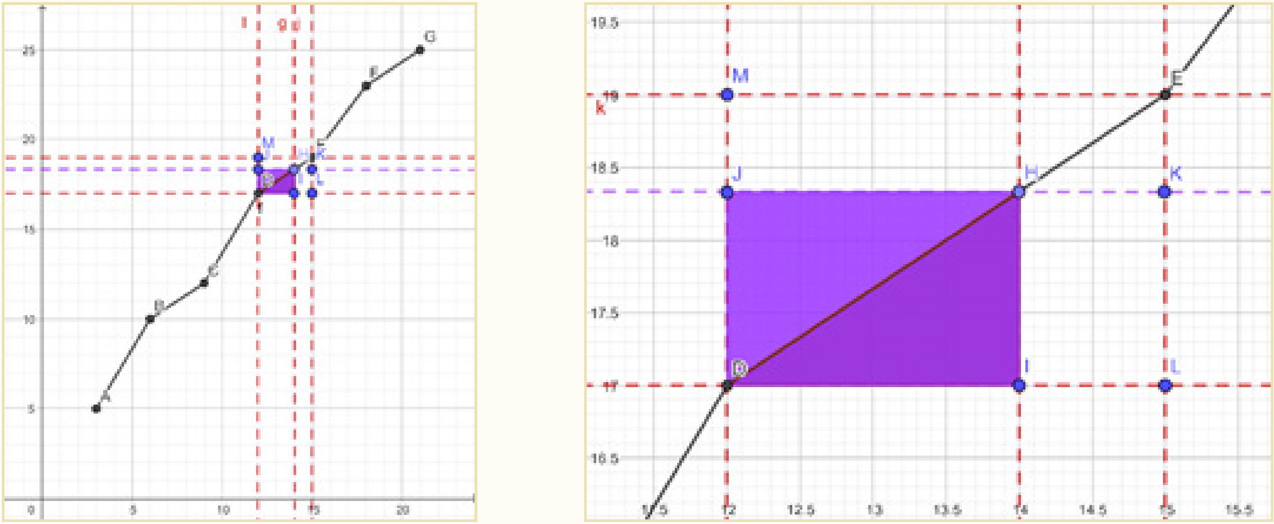
HMA represents the height and capacity data of the loading space in the PST table into a function graph, with the height as the x-axis and the capacity as the y-axis (FT2). When confirmed through an interview, HMA stated that the graphic representation makes finding the relationship pattern of each data pair more straightforward than the table representation. HMA represents the data into a graph with the help of GeoGebra (see Figure 3).

FT3: Finding recursive patterns

Data changes are an essential step in estimating the capacity of the cargo space when the height is 14"m" . HMA found changes in the height and capacity of the cargo space through graphs with the help of GeoGebra (FT3). The interview results confirmed that the graph was a tool for HMA to find a recursive pattern in the form of two similar rectangles. HMA constructed the first rectangular DIHJ with the height data as the length of the rectangle (DI; HJ) and the capacity as the rectangle's width (DJ; IH). HMA constructed the second d rectangle DLEM with the height data as the length of the rectangle (DL; EM) and the capacity as the rectangle's width (DM; LE).

FT4: Finding covariational relationship

Based on the interview results, HMA confirmed that "every three times the height of the loading space increases, the cargo space capacity increases by no more than five (FT4). The relationship is found in the table. The graph shows the relationship more clearly by focusing on the data at a height of 12"m",14"m,and" 15"m" . HMA found a relationship that every time the length of the rectangle increases, the width also increases, forming rectangular patterns that can be seen in Figure 3b (FT4).



(a) (b)

Figure 3 (a) Graph representation (FT2) by HMA; (b) Recursive pattern discovery (FT3) by HMA

FT5: Finding correspondence relationship

HMA finds a correspondence relationship based on the pattern of two similar quadrilaterals. This finding is the basis for HMA to create a general rule (FT5). HMA compares the length (p_1; p_2) and width (l_1; l_2) of the two quadrilaterals to obtain a general rule of linear interpolation marked with a red box in Figure 4. HMA answers that when the height of the loading space is 14m, the capacity is 18,33DWT.

$$\frac{l_2}{l_1} = \frac{p_2}{p_1} \leftrightarrow \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \leftrightarrow y - y_1 = \frac{x - x_1}{x_2 - x_1}(y_2 - y_1)$$

$$\boxed{\leftrightarrow y = y_1 + \frac{x - x_1}{x_2 - x_1}(y_2 - y_1)} \leftrightarrow y = 17 + \frac{19 - 17}{15 - 12}(14 - 12) \leftrightarrow y = 18{,}33$$

Figure 4 General rule by HMA

FT6: Evaluating generalization rule

Based on confirmation through interviews, HMA evaluates the general rule by trying various data in the table. If the calculation results of the cargo capacity obtained do not significantly differ from the known cargo capacity in the table, then HMA considers the general rule suitable for estimating unknown data among known data (FT6).

2. The student with moderate mathematical ability (MMA)

FT1: Identifying the problem

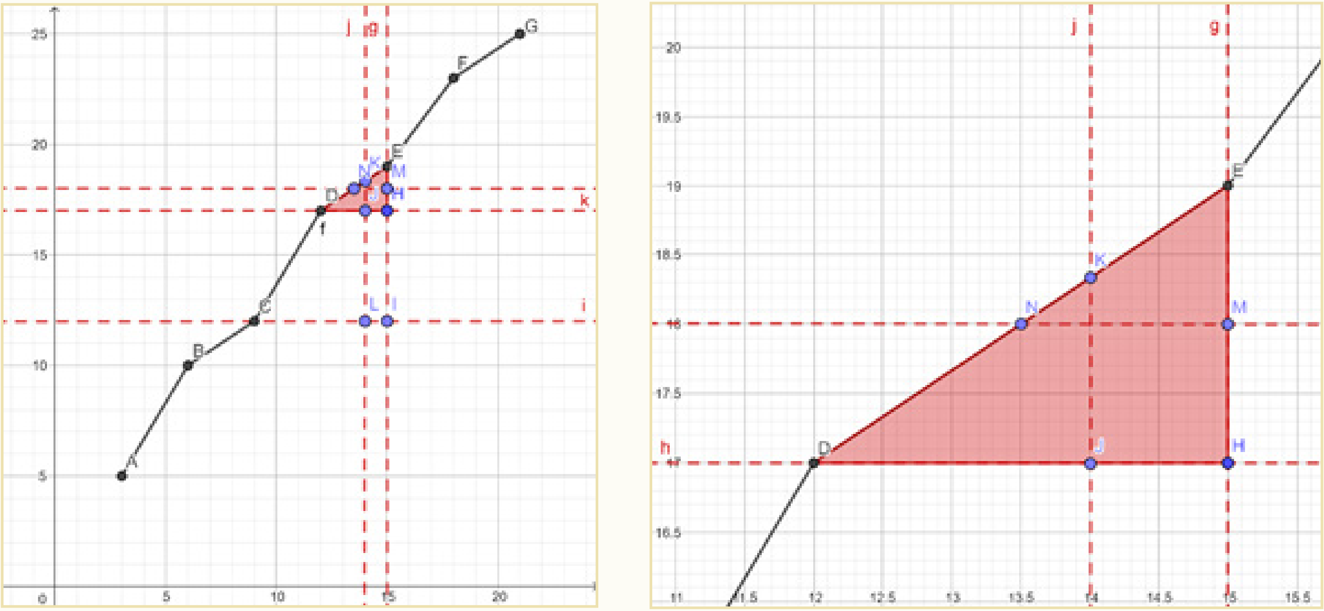
MMA begins to understand the problem by reading the PST as a whole. MMA identifies a relationship between height and tanker load capacity (FT1). MMA states that increasing the height of the tanker's cargo hold increases the tanker's load capacity.

FT2: Representing data

MMA transforms the height and capacity data of the loading space in the table into a function graph with the help of GeoGebra, as shown in Figure 4 (FT2). The height is represented by the x-axis, and the capacity is represented by the y-axis. Based on the interview transcript from MMA, the graphic representation solves the problem of overcoming the difficulty of finding the relationship pattern of each pair of data in the table.

FT3: Finding recursive patterns

MMA found changes in data on the height and capacity of the cargo space through the graph that had been created (FT3). MMA found a recursive pattern in the form of two similar triangles; the first triangle is symbolized by Δ_{DJK}, the second triangle is symbolized Δ_{DHE} (see Figure 5b)



(a) (b)

Figure 5 (a) Graph representation (FT2) by MMA; (b) Recursive pattern discovery (FT3) by MMA

FT4: Finding covariational relationship

MMA had difficulty understanding the relationship between data pairs through tables. MMA found the relationship between each data pair of height and cargo space capacity through graphs by focusing on data at heights of 12m, 14m, and 15m. The relationship between the two quantities found was that each time the base of the triangle increases, the height also increases, which can be seen in Figure 5b (FT4).

FT5: Finding correspondence relationship

MMA compares two similar triangles to find a general rule in estimating the ship's cargo space capacity when the cargo space height is 14m (FT5). The general rule found is a linear interpolation rule marked with a red box in Figure 6 by comparing the base (a_1; a_2) and height (t_1; t_2). Based on the calculation in Figure 6, it is obtained that when the cargo space height is 14m, the cargo space capacity is 18,33DWT.

$$\frac{t_2}{t_1} = \frac{a_2}{a_1} \leftrightarrow \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \leftrightarrow y - y_1 = \frac{x - x_1}{x_2 - x_1}(y_2 - y_1)$$

$$\boxed{\leftrightarrow y = y_1 + \frac{x - x_1}{x_2 - x_1}(y_2 - y_1)} \leftrightarrow y = 17 + \frac{19 - 17}{15 - 12}(14 - 12) \leftrightarrow y = 18{,}33$$

Figure 6 General rule by MMA

FT6: Evaluating generalization rule

MMA evaluates the general rule by zooming in on the graph in GeoGebra so that the capacity of the cargo hold is between 18,3332DWT and 18,3334DWT when the cargo hold's height is 14m (see Figure 7). This capacity is not much different from the calculation results in Figure 6 previously (18,33DWT), so MMA considers the general rule suitable for estimating unknown data among known data.

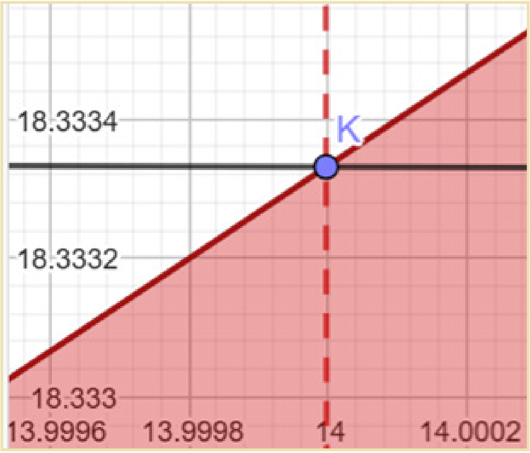


Figure 7 General Rule Evaluation by MMA

3. The student with low mathematical ability (LMA)

FT1: Identifying the problem

LMA reads the question in the problem first before reading the description of the problem. Based on the activity, LMA notices a relationship between the height and the load capacity of the tanker (FT1): when the height of the cargo hold is longer, the load capacity of the tanker is greater. The height of the cargo hold affects the load capacity of the tanker.

FT2: Representing data

LMA needed help estimating the loading space capacity at a height of 14m based on the problem table. The problem table presents data pairs between the height and the loading space capacity. The results of the problem identification were not enough to answer the problem questions, so LMA used GeoGebra to convert the data from the table to a graph (FT2). LMA represented the loading space height data on the x-axis and the loading space capacity data on the y-axis (Figure 8a).

FT3: Finding recursive patterns

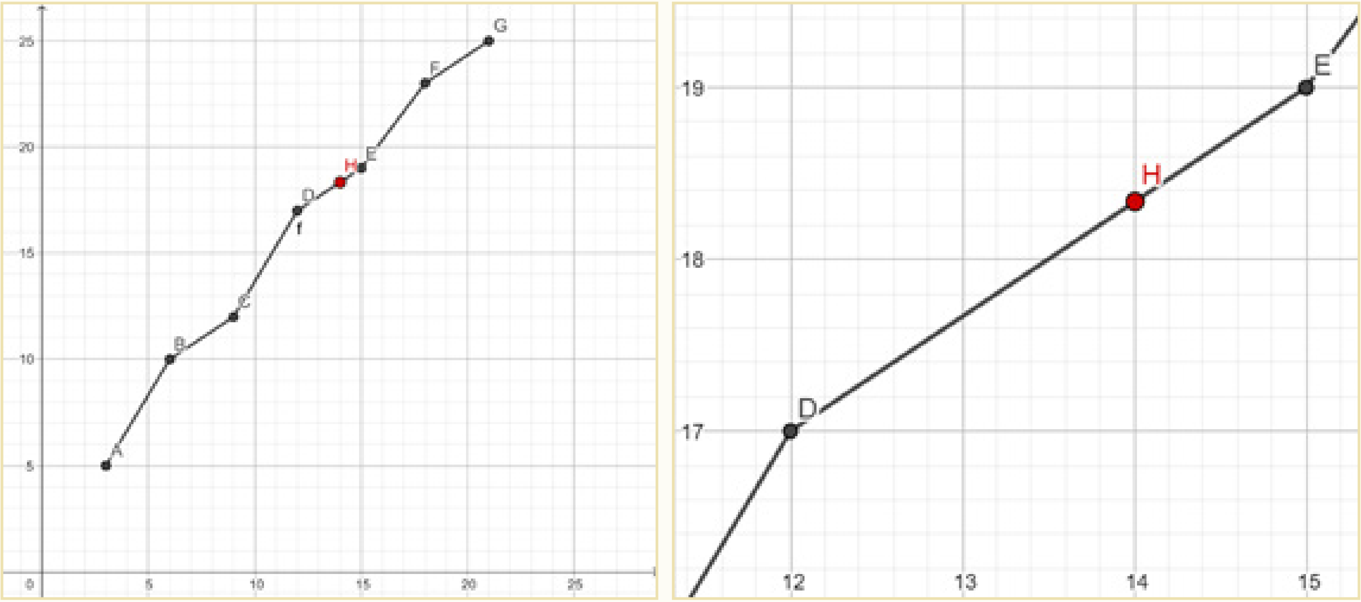
LMA finds changes in height data and load capacity in straight lines between the two closest points on the graph that has been created (FT3). There are six straight lines between the two closest points (see Figure 8a).

FT4: Finding covariational relationship

LMA plots point (14,y) on a straight line between points (12,17) and (15,19). In Figure 8b, LMA finds that the points on the graph are increasing upward (FT4). According to LMA, the increase represents the addition of each ship's height and load capacity.

FT5: Finding correspondence relationship

Based on the straight line between the two points passing through (14,y), LMA adds the two cargo space capacities when 17DWT (y_0) and 19DWT (y_1) and then divide them by two (see Figure 9). LMA does not change the mathematical symbols related to the method used to estimate cargo space capacity based on known data. Based on these calculations, the cargo space capacity is obtained as 18DWT when the cargo space height is 14m, as shown in Figure 9 below.



(a) (b)

Figure 8 (a) Graph representation (FT2) by LMA; (b) Recursive pattern discovery (FT3) by LMA

$$y = \frac{17 + 19}{2} = 18$$

FT6: Evaluating generalization rule

LMA did not evaluate the method used to estimate the ship's cargo space capacity based on known data. Based on the interview transcript, LMA was confident with the method used because the calculation results remained on a straight line between the two closest points. Thus, LMA considered the method appropriate for estimating unknown data among known data.

DISCUSSION

Based on the results of this study, most maritime vocational college students were in category D, which means they achieved three of the six aspects of functional thinking, including identifying problems, representing data, and finding recursive patterns. This achievement indicates the need to involve activities that can develop students' functional thinking processes, such as the use of GeoGebra in solving linear interpolation problems. Several previous studies reported that Geogebra is one of the digital teaching aids successfully used to develop students' functional thinking processes siswa [25; 27]. The success of utilizing GeoGebra is in line with the findings of this study, especially in students with different mathematical abilities. Students with high mathematical ability (HMA) and students with moderate mathematical ability (MMA) achieved five of the six aspects of functional thinking before using GeoGebra to solve problems. After using GeoGebra, HMA and MMA achieved all aspects of functional thinking. HMA and MMA found different general rules for linear interpolation. Furthermore, students with low mathematical ability (LMA) also experienced an increase in the achievement of functional thinking aspects after using GeoGebra. Previously, LMA only achieved three of the six functional thinking aspects. After using GeoGebra, LMA achieved four of the six functional thinking aspects. However, LMA has yet to find a general rule for linear interpolation. This finding indicates that in addition to GeoGebra, differences in mathematical ability also affect differences in the improvement of functional thinking.

Utilizing GeoGebra can improve students' functional thinking because they are involved in interactive visualization. Visualization makes it easier abstraction for students to understand and model real-world situations into function graphs. Visualization supports the functional thinking process because it is a bridge for students to understand patterns and build relationships between two quantities before moving on to higher abstraction steps [13]. The function graph created by students with the help of GeoGebra represents the relationships between the height of the cargo hold and the loading capacity of a tanker ship, which makes it easier for students to understand the concept of linear interpolation functions. Students observe the relationship between the data on the function graph more accurately because they can zoom in, zoom out and perform other interactive movements on the function graph in GeoGebra. The straight line between the two points drawn by students in GeoGebra helps them understand the concept of a straight line as a representation of a linear function. In addition, they can find triangle or quadrilateral patterns by adding auxiliary lines between the two points. Previous research also supports the role of GeoGebra in improving students' functional thinking because it provides them with more opportunities to explore relationships between variables through interactive graphical representations in a more concrete way [25; 28].

Students use GeoGebra to solve linear interpolation problems on PST contains non-routine mathematical tasks that trigger students' involvement in functional thinking processes. This aligns with previous studies using non-routine mathematical tasks to develop students' functional thinking [29; 30]. Non-routine tasks contain two interrelated variables: the height of the cargo hold and the load capacity of the tanker. The load capacity reflects the total load that can be accommodated by the ship. If the load height exceeds the load capacity, the ship can become unstable because the load is not distributed correctly [31]. Ships that lose stability will increase the risk of capsizing and sinking, causing losses [9]. Analysis of the relationship between these variables is critical in shipping planning and preventing ship accidents. This study discovered linear interpolation through graphical representation, which helps estimate the relationship between variables to prevent ship accidents.

Non-routine tasks require guess-and-check, working backwards, and looking for patterns, so they require cognitive processes to complete them, such as functional thinking processes [15]. The process involves problem-solving stages, including analysis, design, exploration, implementation, and verification. In this study, the problem-identifying aspect emerged at the analysis stage because students began to identify problems by recognizing changes in each data set on tankers' height and load capacity. The representing data aspect emerged at the design stage because students designed a strategy based on the characteristics of the problem, namely by changing data pairs in the table into graphs. Students made the graph design because the problem was complex to solve by looking at the table. The finding recursive patterns aspect emerged at the exploration stage because students found changes in the data on tanker height and load capacity based on graphs by trying various solution strategies. The finding of covariational relationships also emerged at the exploration stage because students tried various strategies to find relationships between the data on tanker height and load capacity through graphs to estimate unknown values. In line with Pittalis et al. [18], the exploration stage is known to determine students' success in continuing the process of finding general rules of relationships between quantities. The generalization is found in the correspondence relationships aspect. This aspect appears at the implementation stage because students apply solutions from exploration results by performing calculations so that general rules of linear interpolation are obtained to estimate unknown data values among known data. The evaluating general rule aspect appears at the verification stage because students re-check the correctness of the solutions obtained.

CONCLUSION

In conclusion, the study shows that the use of GeoGebra can improve the functional thinking process of maritime vocational college students in solving problems based on differences in mathematical abilities. GeoGebra facilitates students in the abstraction process to model real-world situations in the form of relationships between the height of the cargo hold and the loading capacity of tankers through interactive visualization into function graphs. Through the function graph, students with high and medium mathematical abilities can find general rules of linear interpolation by comparing the pattern of two similar triangles and the pattern of two similar quadrilaterals. Meanwhile, students with low mathematical abilities do not symbolize the method used to estimate unknown data among known data. The study's findings directly affect learning practices in maritime training professionals by involving non-routine tasks using GeoGebra. However, this study has limitations in the types of tasks with table and graphic representations. Further research is needed to explore other types of tasks that have the potential to improve the functional thinking process in each student with differences in mathematical abilities.

ACKNOWLEDGEMENTS

We thank the Center for Higher Education Funding (Balai Pembiayaan Pendidikan Tinggi) and The Indonesia Endowment Funds for Education (Lembaga Pengelola Dana Pendidikan), which funded this study.

REFERENCES

1. Organisation for Economic Co-operation and Development (OECD), Program For International Student (PISA) 2022 Assessment and Analytical Framework. 2023. [Online]. Available: https://www.oecd-ilibrary.org/education/pisa-2022-assessment-and-analytical-framework_dfe0bf9c-en (accessed 10 October 2024).
2. Wu Z. Effects of Using Problem of The Week in Teaching on Teacher Learning and Change in Algebraic Thinking and Algebra. *ZDM - Mathematics Education*, 2017, vol. 49, no. 2, pp. 203–221, 2017, DOI: 10.1007/s11858-017-0844-x.
3. Taguma M., Makowiecki K., Gabriel F. OECD Learning Compass 2030: Implications for Mathematics Curricula. *In Mathematics Curriculum Reforms Around the World (The 24th ICMI Study)*, 2023, pp. 479-510. DOI: 10.1007/978-3-031-13548-4_33.
4. Vollrath H. J. Search Strategies as Indicators of Functional Thinking. *Educational Studies in Mathematics*, 1986, vol. 17, pp. 387–400. DOI: 10.1007/BF00311326
5. Tarida L., Budiarto M. T., Lukito A. State of the Art of Functional Thinking, Scaffolding, Problem Solving and Self Efficacy (A Systematic Mapping Study). *AIP Conf. Proc*, 2024, vol. 3046, no. 1, DOI: 10.1063/5.0194597.
6. Smith E. Representational Thinking as a Framework for Introducing Functions in the Elementary Curriculum. *Mahwah, NJ: Lawrence Erlbaum Associates/Taylor & Francis Group and National Council of Teachers of Mathematics*, 2008. DOI: 10.4324/9781315097435-6.
7. Frey K., Sproesser U., Veldhuis M. What is Functional Thinking? Theoretical Considerations and First Results of an International Interview Study. *In Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)*, 2022, Brozen-Bolzano.
8. Wei H., Bos R., Drijvers P. An Embodied Approach to Abstract Functional Thinking Using Digital Technology: A Systematic Literature Review. *International Journal of Technology in Mathematics Education*, 2023, vol. 30, no. 2, pp. 75–94. DOI: 10.1564/tme.
9. Huang L. A Mathematical Modeling and an Optimization Algorithm for Marine Ship Route Planning. *Journal of Mathematics*, 2023, vol. 2023, pp.1-8. DOI: 10.1155/2023/5671089.
10. KNKT. Laporan Investigasi Kecelakaan Kapal Laut Tenggelamnya KM. Sinar Bangun 4 Tahun 2018. *Kementerian Perhubungan*, 2018, pp. 1–53 [Online]. Available: http://knkt.dephub.go.id/knkt/ntsc_maritime/Laut/2018/FINAL KNKT-18-06-18-03 Sinar Bangun 4-Final PrintA.pdf (accessed 8 October 2024).
11. Blanton M. L., Kaput J.J. Functional Thinking as a Route Into Algebra in the Elementary Grades. *In Early Algebraization: A Global Dialogue from Multiple Perspectives*, 2011, pp. 5–23, DOI: 10.1007/978-3-642-17735-4_2.
12. Stephens A. C., Fonger N., Strachota S., Isler I., Blanton M. L., Knuth E., Murphy G. A. A Learning Progression for Elementary Students' Functional Thinking. *Mathematical Thinking and Learning*, 2017, vol. 19, no. 3, pp. 143–166. DOI: 10.1080/10986065.2017.1328636.
13. Pitta-Pantazi D., Chimoni M., Christou C. Different Types of Algebraic Thinking: an Empirical Study Focusing on Middle School Students. *International Journal of Science and Mathematics Education*, 2020, vol. 18, no. 5, pp. 965–984, DOI: 10.1007/s10763-019-10003-6.
14. Kriegler S. Just What is Algebraic Thinking? *Mathematics Educator*, 2004, vol. 8, no. 1, pp. 139–151.
15. Schoenfeld A. H. Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics (Reprint). *Journal of Education*, 2016, vol. 196, no. 2, pp. 1–38, 2016, DOI: 10.1177/002205741619600202.
16. Bokau J. R. K., Sulistiana O. Reshaping Curriculum of Indonesian Maritime Education and Training. *International Journal of Social Service and Research*, 2024, vol. 4, no. 6, pp. 1–14, DOI: 10.46799/ijssr.v4i6.809.
17. Pang J.S., Sunwoo J. Design of a Pattern and Correspondence Unit to Foster Functional Thinking in an Elementary Mathematics Textbook. *ZDM - Mathematics Education*, 2022, vol. 54, no. 6, pp. 1315–1331, 2022. DOI: 10.1007/s11858-022-01411-0.
18. Pittalis M., Sproesser U., Demosthenous E., Odysseos E. Enhancing Functional Thinking in Grade 5–6 Students Through a Dynamic Mathematics Intervention Program. *Education and Information Technologies*, 2024, pp. 1-33. DOI: 10.1007/s10639-024-12865-y.
19. Jamaliyatul N. A. A. H., Badyalina B., Mokhtar N. A., Rambli A., Zubairi Y. Z., Ghapor A. A. Modelling Wind Speed Data in Pulau Langkawi With Functional Relationship. *Sains Malaysiana*, 2023, vol. 52, no. 8, pp. 2419–2430, DOI: 10.17576/jsm-2023-5208-18.
20. Chapra S. C., R. P. Canale, Numerical Methods for Engineers, vol. 1221. New York: Mcgraw-hill, 2011.
21. Wahab M. A. Interpolation and Extrapolation. In Proceeding Topics in System Engineering. *Winter Term*, 2017, pp. 1–6. DOI: 10.1201/9781315120195-4.
22. Schoenfeld A. H. Mathematical Problem-Solving. Academic Press, 1985. DOI: 10.1007/BF00305624.
23. Rott B., Specht B., Knipping C. A Descriptive Phase Model of Problem-Solving Processes. *ZDM - Mathematics*

Education, 2021, vol. 53, no. 4, pp. 737–752. DOI: 10.1007/s11858-021-01244-3.
24. Lindenbauer E. A Digital Worksheet for Diagnosing and Enhancing Students' Conceptions in Functional Thinking. *Mathematics Education in the Digital Age (MEDA),* 2020, pp. 247–254.
25. Günster S. M., Weigand H. G. Designing Digital Technology Tasks for the Development of Functional Thinking. *ZDM - Mathematics Education*, 2020, vol. 52, no. 7, pp. 1259–1274, DOI: 10.1007/s11858-020-01179-1.
26. Schoenfeld A. H. Problem Solving from Cradle to Grave. In *Annales de Didactique et de Sciences Cognitives*, 2006, vol. 11, pp. 41–73.
27. Vainshtein Y. v., Saryglar S. V. Album of Animated Drawings as an Innovative Means of Teaching Algebra in the Digital Environment of Bilingual Students. *Perspektivy Nauki i Obrazovania*, 2024, vol. 69, no. 3, pp. 319–336. DOI: 10.32744/pse.2024.3.19.
28. Lichti M., Roth J. Functional Thinking—A Three-Dimensional Construct? *Journal fur Mathematik-Didaktik*, 2019, vol. 40, no. 2, pp. 169–195. DOI: 10.1007/s13138-019-00141-3.
29. Aguilar J., Telese J. A. Mathematics Understanding of Elementary Pre-service Teachers: The Analysis of Their Procedural-fluency, Conceptual-understanding, and Problem-solving strategies. *Mathematics Teaching Research Journal*, 2018, vol. 10, no. 3–4, pp. 24–37.
30. Pang J., Leena L., Sunwoo J. Task Development to Measure Functional Thinking: Focusing on Third Graders' Understanding. *The Korean Society of Educational Studies in Mathematics - Journal of Educational Research in Mathematics*, 2022, vol. 32, no. 3, pp. 351–372, DOI: 10.29275/jerm.2022.32.3.351.
31. Mycroft K., Sriraman B. Mathematics in the Maritime. In *Handbook of the Mathematics of the Arts and Sciences*, 2021, pp. 1593–1612, DOI: 10.1007/978-3-319-57072-3_137.

Authors

Luthfiana Tarida
(Surabaya, Indonesia)
Doctoral Program Student, Mathematics Education Study Program
Universitas Negeri Surabaya
***
Mathematic Lecturer, Departement of Deck Officer Program
Akademi Maritim Nusantara
E-mail: luthfiana.21012@mhs.unesa.ac.id
ORCID ID: 0000-0003-1879-9829

Mega Teguh Budiarto
(Surabaya, Indonesia)
Professor in Mathematics Education Study Program
Universitas Negeri Surabaya
E-mail: megatbudiarto@unesa.ac.id
ORCID ID: 0000-0002-9052-3141

Agung Lukito
(Surabaya, Indonesia)
Doctor in Mathematics Education Study Program
Universitas Negeri Surabaya
E-mail: agunglukito@unesa.ac.id
ORCID ID: 0000-0003-1277-5834

Author's contribution

Luthfiana Tarida: Conceptualization, Writing - Original Draft, Methodology, Visualization, Funding acquisition
Mega Teguh Budiarto: Conceptualization, Writing - Review & Editing, Formal analysis, Methodology, Validation and Supervision
Agung Lukito: Conceptualization, Writing - Review & Editing, Formal analysis, Methodology, Validation and Supervision

The author's declared no conficts of interest