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О предметном содержании математических олимпиад школьников

Введение. Актуальность исследования обусловлена необходимостью усовершенствования процедуры отбора содержания математических олимпиад школьников. *Проблема исследования* заключается в определении единых требований к отбору содержания задач школьных математических олимпиад, с учетом многообразия форм и видов математических олимпиад.

Для достижения цели применялись *методы*: сравнительный анализ содержания олимпиадных задач по математике в Кыргызстане и странах зарубежья, анализ методической литературы по теории и практике решения олимпиадных задач по математике, программных документов, протоколов олимпиад. Особое внимание уделяется изучению и обобщению результатов олимпиад с участием школьников Кыргызской Республики.

Результаты исследования. Содержание олимпиадных заданий по математике должно быть направлено на возможность демонстрации участниками олимпиады творческого подхода, включая наличие переменного диапазона ответов. Задачи должны включать объективные и универсальные образовательные действия в единстве. Критерии оценки олимпиадных работ различаются в зависимости от уровня и статуса олимпиады, возраста ее участников. К решениям заданий финального этапа олимпиады предъявляются самые строгие требования, поэтому, необходимо обучать участников, помимо предметных знаний, свободно владеть математическим языком и техникой аргументированных рассуждений. Школьники Кыргызстана принимают участие в более 10 математических олимпиадах международного статуса. Несмотря на то, что на олимпиадах среди республик постсоветского пространства, наши школьники занимают II и III места, все же их знания не соответствуют олимпиадам международного уровня. Для решения этой проблемы необходимо приблизить республиканские олимпиады к международным нормам, разработать программу олимпиадной математики, соответствующую уровню международных математических олимпиад, обучить профессиональных тренеров внутри страны, которые смогут качественно подготовить школьников к участию в IMO, ужесточить отбор участников на международные олимпиады.

Заключение. Соответствие задач содержанию олимпиады каждого уровня способствует ее понятной организации, объективности и прозрачности. Для объективного определения победителей олимпиады необходима точная оценка характеристик олимпиадных задач, таких как: относительная сложность, дифференцирующая способность, обоснованность задания, соответствие уровню подготовки.

Перспективы. В исследовании рассмотрено содержание традиционных форм математических олимпиад школьников, при этом остается возможность дальнейшего исследования аспектов отбора содержания дистанционных и открытых форм олимпиад.

Ключевые слова: олимпиада, математика, содержание, задача, решение, критерии оценки, сложность задачи

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About the subject content of Mathematical Olympiads for schoolchildren

Introduction. The relevance of the study is due to the need to improve the procedure for selecting the content of Mathematical Olympiads for schoolchildren. *The problem of the study* is to determine uniform requirements for selection of the content of the problems of school Mathematical Olympiads, taking into account the variety of forms and types of Mathematical Olympiads.

Materials and methods. To achieve the goal, a comparative analysis of the content of Olympiad problems in mathematics, analysis of methodological literature on the theory and practice of solving Olympiad problems in mathematics, program documents, Olympiad protocols were used in Kyrgyzstan and foreign countries. Special attention is paid to the study and generalization of the results of Olympiads with participation of schoolchildren of the Kyrgyz Republic.

Research results. The content of Olympiad problems in mathematics should be aimed at allowing participants to demonstrate a creative approach, including the presence of a variable range of responses. Problems should include objective and universal educational actions in unity. The criteria for evaluating the Olympiad works differ depending on the level and status of the Olympiad, the age of its participants. The most stringent requirements are imposed on the final stage of the Olympiad, so it is necessary to train participants, in addition to subject knowledge, to be fluent in the mathematical language and technique of substantiated reasoning. Schoolchildren of Kyrgyzstan take part in more than 10 international mathematical Olympiads. Despite the fact that our students take II and III places at the Olympiads among the republics of the former Soviet Union, their knowledge does not correspond to the international Olympiads. To solve this problem, it is necessary to bring the republican Olympiads closer to international standards, develop a program of Olympiad mathematics that corresponds to the level of international mathematical Olympiads, train professional coaches in the country who will be able to prepare students for participation in IMO, tighten the selection of participants for international Olympiads.

Conclusion. Compliance of problems with the content of the Olympiad at each level contributes to its clear organization, objectivity and transparency. To objectively determine the winners of the Olympiad, an accurate assessment of the characteristics of the Olympiad problems is necessary, such as: relative complexity, differentiating ability, validity of task, and compliance with the level of training.

Prospects. The study examines the content of traditional forms of Mathematical Olympiads for schoolchildren, while it remains possible to further study the aspects of selecting the content of remote and open forms of Olympiads.

Keywords: Olympiad, mathematics, content, problem, solution, evaluation criteria, task complexity

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Introduction

Traditionally, the Olympiad is an organizational form of work with gifted children. Gifted children are distinguished by high academic performance, wide erudition; teachers have also noted such qualities as patience and diligence in work [1]. When solving the Olympiad problem, students show signs of mathematical abilities: «The ability to generalize; logical and formalized thinking; flexibility, depth, systematic, rational and solid reasoning; mathematical perception and memory» [2]. The above qualities are successfully formed and developed during the preparation and participation in the Olympiads. For example, schoolchildren of the People's Republic of China took the first places 13 times before 2006, and 6 times from 2009 to 2019 in the International Mathematical Olympiad (IMO) in the team competition, winning the strongest countries in the world like USA, South Korea, Russia, Japan, Vietnam, Taiwan, and Singapore. Most of its participants were awarded gold medals. Singaporean mathematicians Xiong Bin, Lee Peng Yee believe, that such a success is due to three factors: hard work and perseverance of the students themselves; professionalism and efforts of school teachers and national trainers; a feature of the educational system in China that focuses on the preparation of basic skills in the field of science education [3, p. 7].

The desire to achieve high results is one of the functions of the Olympiad: «To win, you need to know the subject more profoundly. In the process of preparing for a regional tour, the desire to win among students grows stronger, becomes dominant - that is why the appeal procedure is more important at Olympiads» [4]. When passing the appeal, the participant should be able to see the final score received by him or her for performing the Olympiad work, as well as the points that he or she received according to each criterion. Researchers Van Dooren W., Verschaffel L. & Onghena P. note that we need assessment criteria that would help us choose the right formal method to solve the problem at hand [5].

Coaches of the national teams in mathematics of Kyrgyzstan, Kazakhstan, and Russia also focus on the nature of the mistakes made and evaluate the degree of advancement of the participant in the Olympiad in solving the problem; when evaluating the full solution in points, only specific, clearly formulated, confirmed statements that lead to a complete solution are taken into account. In the Kyrgyz Republic, the regional stage of the Republican Olympiad is held for students of grades 10-11 of schools in the regions and cities of Bishkek and Osh after the previous school and district stages.

The tasks of the Olympiad are developed according to the **principles**:

- assignments should be aimed at identifying highly motivated students with learning skills in a particular theoretical field, the ability to apply knowledge in new conditions, analyze, evaluate various approaches to solving tasks or problems, finding non-standard solutions, as well as the ability to argue their point of view;
- assignments can allow a wide range of answers, the formulation of several hypotheses, various arguments and other possibilities for students to be creative;
- tasks should have clear assessment criteria (the independence and consistency of thinking, the knowledge of information for formulating arguments, the skills of proof and argumentation, the novelty of the solution are evaluated);
- assignments should be aimed at assessing the student's skills to independently apply their knowledge in practice, set tasks and solve them in new conditions [6].

The tasks of the mathematical Olympiad are aimed at identifying the participants' skills and abilities to apply knowledge in the new conditions, analyze, evaluate various approaches to solving problems or problems, find non-standard solutions, the ability to argue their point of view. An objective assessment of the Olympiad work is ensured by the conformity of the evaluating factors, the features by which an unambiguous assessment is made, and the use of adequate meters. Assessment of written / practical/ oral assignments should be based on clearly defined criteria.

Literature review

The content of mathematical Olympiads is of various levels, from municipal to international, various methods for solving Olympiad problems in mathematics are presented in the works of Russian authors [7; 8], Kazakh authors [9], as well as other foreign researchers [10-14].

Aspects of the relationship between mathematical creativity and mathematical abilities are investigated in the work Kattou M., Kontoyianni K., Pitta-Pantazi D. & Christou C. [15]. Various methods for solving Olympiad problems in mathematics are presented Agahanov N.H. & Podlipsky O.K. [4], Losada M.E. [16] and others.

Researches Szetela W. & Nicol C. [17] and Veilande I., Ramana L. & Krauze S. [18] are devoted to evaluating the solution of problems in mathematics. An article by Lazarev V.A. & Khaybullin R.Y. [19] is devoted to the method of assessing the relative difficulty and differentiating ability of tasks proposed within the framework of a single test, competition, subject Olympiad. The problem of developing criteria for analyzing student answers in expert systems for monitoring and evaluating knowledge is considered in the article Golovachyova V.N., Tomilova N.I. & Abildaeva G.B. [20].

The research does not sufficiently reflect the influence of the content of the Olympiad tasks on the effectiveness and quality of mathematical Olympiads. Considering the need for uniform requirements for the program, the purpose of the research is to consider the subject content and criteria for assessing the solution of problems of mathematical Olympiads with an emphasis on Olympiads with the participation of schoolchildren of the Kyrgyz Republic.

Materials and methods

To solve this goal, **research methods** were used:

1. a theoretical analysis and comparison of the identified approaches to the study of the criteria for assessing Olympiad problems in mathematics in Russia, Kyrgyzstan, Kazakhstan;
2. analysis of methods for solving Olympiad problems in mathematics;
3. analysis of the results of the regional, city and republican stages Mathematical Olympiad for schoolchildren 2019 in Kyrgyzstan;
4. the study and generalization of pedagogical and methodological experience in organizing subject Olympiads of schoolchildren, monitoring the process of evaluating Olympiad works.

Results and discussion

1. On the content of problems of the mathematical Olympiad

The final stage of the republican Olympiad is the launching pad for participation in international Olympiads, so there is a need to pay attention to the content of the tasks included in the package of Olympiad tasks. The analysis of specific Olympiads allowed the authors Lazarev V.A. & Khaybullin R.Y. of the study [19] to conclude the presence of insignificant tasks in the package of the proposed Olympiad tasks that are inappropriate to achieve the goal of the Olympiad, and an excessive number of tasks are often offered at Olympiads. Falk de Losada Mary shares her thoughts on the problem of developing assignments for the various stages of the Colombian Olympiad in mathematics and believes that the annual results demonstrate how much the participant has advanced in the development of his mathematical thinking [21].

In school education, such large sections of mathematics as trigonometry, geometry and combinatorial analysis is currently ignored [22]. At the same time, authors pay attention to the need to teach schoolchildren special methods for solving problems of modern geometric Olympiads [10; 12; 16].

The practice of conducting mathematical Olympiads in Kyrgyzstan shows that students are better prepared for problems in arithmetic, algebra and mathematical analysis, to a lesser extent in geometry. In the study of Sal'kov N.A., Vyshnyepolskiy V.I., Aristov V.M. & Kulikov V.P. have noted the prevailing situation in Russian school education: «... it's not enough to know school geometry, which is currently almost not taught in schools - not only spatial imagination is necessary, but at least the rudiments of heuristic thinking» [23].

Researchers point to the role of geometry in the development of spatial thinking and the skills of evidence-based reasoning: «The school course of geometry can pursue several goals, among which I will name two: the development of imagination, on the one hand, and the development of the ability to accurately formulate one's thoughts, on the other. Where else, except in geometry lessons, can a person learn to establish the indispensable truth of judgments logically inferred from fundamental facts that are not in doubt?» [7, p. 407], «Geometric thinking ... is a combination of spatial thinking, involving the handling of spatial images, and logical thinking, aimed at establishing the appropriate relationship between these images» [24]. Considering the role of school geometry in the system of subject Olympiads, we concluded that the introduction of geometric material in the course of preparing students for participation in mathematical Olympiads at all levels, developing their mental abilities, provides comprehensive mathematical training for schoolchildren [25].

The content of the largest international mathematical Olympiads annually includes at least one task of combinatorial content because their solution requires a high level of creative thinking [13]. The share of combinatorial, number-theoretic and other problems does not fall below 50% of the total number of tasks of mathematical competitions popular among Latvian schoolchildren [26]. The tasks of the International Mathematical Olympiad (IMO) include such sections of mathematics as combinatorial analysis, geometry, number theory; special attention is paid to evidence [11]. Thus, despite the process of humanization taking place in modern education, and, as a consequence, the reduction in the share of exact disciplines in secondary education curricula in many countries of the world, the content of mathematical Olympiads adheres to traditionally high standards.

Each year, the selection of the content of the tasks of Olympiads is carried out based on an analysis of the results of the Olympiad of the previous year, which shows that high academic performance does not guarantee success [27; 28]. In the formation of the student's desire and ability to solve problems, researchers note the characteristics of the problem: usefulness, importance, interest, and weight. And considering that at the final stage of the Republican Olympiad, applicants are selected for representing the republic at the international level, then the set of Olympiad tasks of the 4th stage, consisting of 6 problems, as a rule, include two geometric and one combinatorial problem.

2. Evaluation criteria for solving Olympiad problems in mathematics

The paramount importance of determining clearly defined criteria for assessing Olympiad tasks in identifying the winners of the Olympiad is indicated by the following facts: «Criteria-based assessment, as a rule, allows you to objectively resolve all disputes (if any) regarding the marks received» [29]. And also: «The correct, competent determination of the evaluation criteria (evaluating factors), indicators (features by which an unambiguous assessment is made), the use of adequate meters (tools with which to evaluate: questionnaires, tests, observation protocols) are the key to a correct assessment any activity, method» [30].

The specifics of the Olympiad problems imply that a creative component must be observed in its solution. Among the evaluation criteria for creative problem solving, R. Harris singles out brevity, rationality, simplicity, and grace [31].

In Olympiads of different levels, designed for students of different ages, the criteria for evaluating Olympiad tasks are also varying flexible. One of the oldest and most prestigious international mathematical Olympiads for schoolchildren – IMO is distinguished by its strict selection of winners and prize winners, which allows identifying the strongest participants. And in the Olympiads for junior schoolchildren of the city (regional) stage, the jury members evaluate not the quality of the solution to the problem, but the degree of its understanding, since students of this age still do not know how to formulate and draw up their decisions in writing. But there are common principles for assessing Olympiad tasks, which are adhered to by the jury of Olympiads of all levels. For example, Olympiad tasks have different solutions; the assessment of the problem should not depend on the volume or rationality of the solution. Nevertheless, I believe that while teaching mathematics to schoolchildren, including preparing students for mathematical Olympiads, it is nevertheless necessary to teach schoolchildren the ability to find the optimal solution.

The research results have shown that future teachers of mathematics prefer to use a more complex algebraic method of solution, with the possibility of rational arithmetic solutions, and the methods of solving future teachers of elementary grades were diverse [5]. Perhaps this suggests that the correct setting and the absence of templates during training contributes to the education of rationality and non-standard thinking of the student. Our opinion is confirmed by the author of the training course on methods for solving Olympiad problems in mathematics, held in Singapore. Xu Jiagu focuses on the fact that mathematical Olympiads are a system of development of mathematical advanced education, i.e. it is more than just training in solving Olympic problems [14].

Often, the assessment of students' Olympiad performance, which is determined by the jury of the Olympiad in points, turns out to be subjective, due to the fact that the Olympiad tasks cannot be formalized; therefore, it is difficult to formulate a universal criterion for their objective assessment. The authors have determined that «even for educational school tasks, the correlation coefficient between expert and statistical estimates may be less than

0,5» [19]. In order to objectively identify the winners and prize winners of the Olympiad, it is necessary to determine the optimal ratio of the complexity of the problem and the points awarded for its correct solution.

Being on the jury of city and regional mathematical competitions for 25 years, we concluded that an objective assessment of the solution of the Olympiad problem in mathematics requires changes in the structure of the task. Olympiad tasks have to be composed so that:

- the student has to understand what exactly is assessed in each task to gain experience in the self-reflection of educational and cognitive activities;
- it is advisable to do the tasks in multi-stage: each task should contain several questions (tasks) connecting it with the subsequent task and requiring additional information, clarifying or correcting actions.

This is necessary to give in each task situation not an elementary conclusion in the form of “solved - not solved”, but to differentiate the assessment:

a) if the task is complete, then it can be assumed that the student understands, knows and knows how to apply knowledge, that is, he or she is competent in this matter (at the level at which the formation of competence is assessed);

b) if the first, simpler stages are completed, then it can be assumed that the level of competence formation is lower than the one for which this question is intended.

The task should not only evaluate the results already achieved, but also focus on further achievements.

- Tasks should provide an assessment of the level of mastery of the system of funds necessary and sufficient for a successful implementation of educational activities.
- Tasks should be designed so that the appropriate means of activity, the assimilation of which is provided in the process of solving the problem, act as a direct learning product.

Testing the solution of the Olympiad problems involves the use of two types of rating scales: quantitative and qualitative. So, in the republican Olympiads of Kyrgyzstan and Russia in mathematics, 7-, 10-point systems for assessing problem-solving are traditionally used, there is an estimate of the complexity of the problem according to a 30-point system. In the Moscow Mathematical Olympiads (MMO), the practice of using the characters +, +., -, -., +/2, \pm, \mp, !, 0 for controlling checking the papers [7; 32]. In the open mathematical Olympiads of Latvia to assess the levels of mathematical competencies of students a special system is used, including codes for each step of solving am12, ap10, etc [18]. In Kazakhstan Olympiads, the A, B, C letter designations are used [9]. At the International Mathematical Olympiad, a 6-level gradation of problem complexity is accepted, table 1:

Table 1

Levels of difficulty of problems in the International Mathematical Olympiad

Task gradation	5+	4-5	3-4	2-3	1-2	0-1
Conditional colour	turquoise	green	yellow	orange	pink	red
Level of difficulty	Very simple	Simple	Medium difficult	Harder than medium	Difficult	Very complex

Note:

The colorgramme of the tasks, according to the level of difficulty are found in the short lists of IMO

Researchers point out that in some cases, a student with good mathematical intuition, without solving a problem, can guess the correct answer [21]. In such a situation, Russian trainers «only the guessed correct answer without explanation in a number of problems in which the answer is chosen from two options, for example, in problems with the question: is it true ..., can it ... or does it exist ...» suggest to estimate at 0 points [4].

The place awarded to the participant at the Olympiad is determined based on the rating of points received. Consideration of the appeal is carried out with the participation of the participant and the teacher of the relevant subject. Therefore, we consider it important that, as a result of the criteria-based assessment, the participant has the opportunity to see the final score received by him for performing the Olympiad assignment, as well as the points that he/she received according to each criterion.

Traditionally, in the republican Olympiad of schoolchildren of the Kyrgyz Republic, all tasks are evaluated, regardless of the degree of difficulty, based on a given number of points, using a quantitative scale. So, in previous years, the complete and correct solution of the Olympiad task in mathematics was estimated at 7 points. Summarizing the personal experience of working in the commission at school mathematical Olympiads in Osh, we consider the 7-point assessment to be imperfect since at each stage of the solution it is necessary to take into account the recommended correspondence of the correctness of the solution and the points set. Based on this, in 2017, in order to reduce subjectivity in assessment, we proposed introducing a 10-point system of criteria for evaluating Olympiad assignment in mathematics. As a result, in 2019 at the regional mathematics Olympiad, tasks have been evaluated according to established criteria on a 10-point scale, table 2:

Table 2

Criteria for a 10-point assessment of the solution of a problem in mathematics [29]

Points	Criteria for the assessment of the Olympiad problem in mathematics
10 p.	Complete right decision with theoretical justification
9 p.	The right solution. There are minor flaws that do not affect the result
7-8 p.	The solution is generally correct, but it contains a number of errors, or not considered individual cases
5-6 p.	The decision has not been finalized, but the progress is in the right direction
3-4 p.	Additional statements are proved that help in solving the problem, but the problem as a whole is not solved
1-2 p.	The answer is correct, but there is no solution
0 p.	Wrong decision, no progress
0 p.	No solution

Thus, the wording of the criteria for evaluating the solution shows the degree of advancement of the student in solving the problem.

The verification procedure of the works showed that the decisions of the winners of the Olympiad are close to the expert method of solution. For example, when solving a problem and searching for a quantity, it is necessary to perform certain actions:

1. Calculate the value.
2. Conclude the change in value.
3. Calculate the number of steps in the solution process using mathematical formulas.
4. Calculate the change in magnitude using mathematical formulas.
5. Write a mathematical calculation formula.

6. Calculate the result.
7. Justify the immutability of the result.

3. Participation of schoolchildren of Kyrgyzstan in the republican Olympiad in mathematics in 2018 and 2019

The characteristic features of the participation of schoolchildren in the Olympiad movement of the Kyrgyz Republic, the structure, the results of various stages of the republican Olympiad of schoolchildren, other types of mathematical competitions, Olympiads in mental arithmetic have been described in [33; 34].

The organization of the republican Olympiad for schoolchildren of the Kyrgyz Republic goes through 4 stages. The Republican Olympiad for schoolchildren starts in October each year. As in the previous year, an independent organization, the Center for Assessment in Education and Teaching Methods, specializing in science-based, independent testing in Kyrgyzstan, took part in the development of the Olympiad tasks, organization and conduct of the Olympiad. 165 trained administrators were sent to each region of the republic, whose duties included the delivery of Olympiad tasks, administration and coordination of the Olympiad in each subject. In Osh, a commission of 60 people worked, including 9 representatives of the organizing committee of the Olympiad, who ensured objectivity in assessment and judgment.

In the subject of mathematics, 7 students won prizes: 2 students – 1st place, 2 students – 2nd place, 1 student – 3rd place, 2 students – 4th place. First place in the team standings, took the students of the city of Osh. Compared with the Olympiads of previous years, there was an increase in the number of winners from the regions of Kyrgyzstan. Figure 1 shows the distribution of prizes in the subject of mathematics.

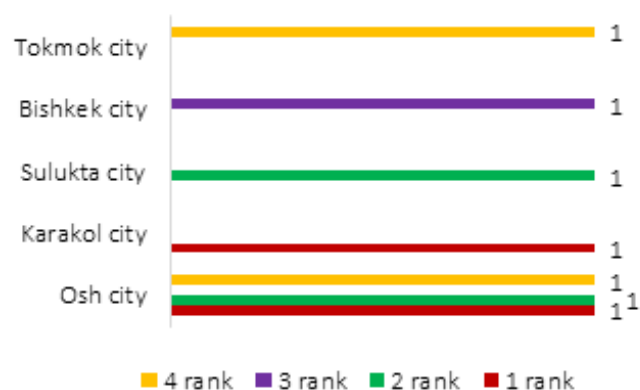


Figure 1 Results of the 2019 math Republican Olympiad

4. Participation of the Olympic reserve of Kyrgyzstan in international mathematical Olympiads

In addition to Olympiads in mathematics, Kyrgyz Republic schoolchildren take an active part in international Olympiads in mental arithmetic, the International Olympiad of «Laboratory for preparation of talents», as well as in other subjects of the natural science cycle: physics, computer science, chemistry and biology [34].

Based on the opinion of Olympiad organizers that a victory at the Olympiad is an indicator of a significant result and a high level of training of schoolchildren, we will consider the results of the Kyrgyz Republic team in Olympiads in mathematics that have international status. At the XVI International Zhautykov Olympiad in mathematics, computer science and physics,

held in Kazakhstan in January 2020, the Kyrgyz Republic national team won 3 bronze medals and diplomas in mathematics. In the VI Iranian 2019 Olympiad in distance geometry, held in Russia, the Kyrgyz Republic students were awarded 1 silver, 1 bronze medals and diplomas of the 4th degree. In the International 2018 Olympiad of youth, held in our capital Bishkek, more than a hundred senior high school students of the republic competed in 16 subjects, including mathematics and physics. At the 2019 International Olympiad organized by the National Research University of Russia «Higher School of Economics», Kyrgyz Republic high school students received 30 diplomas and prizes.

In one of the most prestigious Olympiads in the world - the International Mathematical Olympiad (IMO), our country first took part in 1993. For 27 years, the Kyrgyz Republic students at IMO won 12 medals: 1 silver and 11 bronze. However, at the LX International Mathematical Olympiad in 2019, held in the UK, the Kyrgyz Republic team of 6 participants did not receive awards, taking the 91st place out of 112.

We will consider the complexity of IMO tasks since 1993. Note that the complexity of the tasks refers to the arithmetic mean of all the points scored by the participants in this task. As we already mentioned, the sections of mathematics: algebra, combinatory, geometry, number theory, were distributed in the set of tasks as follows: tasks P1, P2, P4, P5 had contents in one of these sections, problems P3, P6 had contents in two of them. Therefore, two sections are presented in tasks twice. Each task in the IMO is rated at 7 points and is considered solved if the participant scores 5-7 points. For example, in 2019, the distribution of the sections of mathematics and the complexity of the tasks were as follows, table 3:

Table 3

Distribution of sections in the content of tasks and the complexity of the tasks of IMO
2019

Task	Content of tasks of the section of mathematics	Complexity, in points
P1	Algebra (functional equation)	5,18
P2	Geometry (classical planimetry)	2,4
P3	Combinatorics (logical)	0,57
P4	Number Theory (+ inequalities)	3,74
P5	Combinatorics (recurrence relations)	3,57
P6	Geometry (Algebraic)	0,4

Next, we will consider the results of performances of the Kyrgyz Republic students in IMO, table 4.

The right side of the table shows the number of tasks of each difficulty level solved by Kyrgyz Republic students. We see that over the entire history of the participation of the Kyrgyz Republic in IMO, tasks of levels 0-1, 1-2 and 2-3 have been solved only 15 times, which allowed us to earn 12 medals. Tasks of level 3-4 have been solved 7 times.

The analysis of the points awarded for all tasks has led us to the conclusion: to get a silver medal, you need to score more than 20 points for solving problems of levels 3-4 and 2-3, that is, solve at least two sections of mathematics from the Olympiad program at level 2.5 points, and the remaining sections at 4 points. To win a bronze medal, it is necessary to solve at least one section at the level of 2.5 points.

The results of international Olympiads show that despite the fact that our students show average results at Olympiads held among the republics of the Commonwealth

of Independent States, the situation at the international level is worse, as can be seen from the IMO results. One of the problems of this state of the Kyrgyz Republic schoolchildren's Olympiad is that, on the one hand, an accessible environment is formed with the maximum enrollment of students in the school stage of the Olympiad in mathematics in all schools of the country, but on the other hand, there is no systematic professional development covering all teachers of mathematics to prepare students to Olympiads. In addition, we do not have professional trainers in the country who are able to qualitatively prepare the selected IMO students; therefore, we need to think about their qualifications and training. Thus, it is necessary to strengthen the training of both participants and teachers of the Olympic reserve, to conduct a better selection for international Olympiads.

Table 4

The Kyrgyz Republic results in IMO in the period of 1993-2019 [35]

Years	Points received for solving problems						Total	Points needed to get a medal			Rewards of the Kyrgyz Republic schoolchildren	Number of problems solved for each difficulty level						Sum
	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆		B	S	G		5+	4-5	3-4	2-3	1-2	0-1	
2019	5,18	2,4	0,57	3,74	3,57	0,4	15,86	17	24	31		1	0	0	0	0	0	1
2018	4,93	2,95	0,28	3,96	2,7	0,64	15,46	16	25	31	-	0	4	0	0	0	0	4
2017	5,94	2,3	0,04	5,03	0,97	0,29	14,57	16	19	25	2 B	10	0	0	0	0	0	10
2016	5,27	2,03	0,25	4,74	1,68	0,81	14,78	16	22	29	-	3	1	0	0	0	0	4
2015	4,31	1,36	0,65	4,79	1,51	0,36	12,98	14	19	26	-	0	1	0	0	0	0	1
2014	5,35	2,97	0,51	5,19	1,71	0,34	16,07	16	22	29	-	3	0	0	0	0	0	3
2013	4,11	2,53	0,79	5,44	2,45	0,39	15,71	15	24	31	1B	3	0	0	0	0	1	4
2012	5,63	2,55	0,41	3,77	1,66	0,34	14,36	14	21	28	-	6	0	0	0	0	0	6
2011	5,35	0,65	1,06	4,07	3,26	0,32	14,71	16	22	28	-	1	0	0	0	0	0	1
2010	5,45	2,58	0,47	5,35	0,93	0,37	15,15	15	21	27	1S + 1B	6	0	0	2	0	0	8
2009	4,8	3,71	1,02	2,92	2,47	0,17	15,09	14	24	32	-	0	0	3	1	0	0	4
2008	4,98	2,56	0,8	4,4	2,08	0,26	15,08	15	22	31	-	0	2	0	0	0	0	2
2007	3,38	2,52	0,3	5,68	1,9	0,15	13,93	14	21	29	1B	3	0	0	1	1	0	5
2006	5,61	1,83	0,66	5	1,18	0,19	14,47	15	19	28	-	2	0	0	0	0	0	2
2005	2,61	3,05	0,91	3,76	2,17	1,35	13,85	12	23	35	2B	0	0	2	3	0	0	5
2004	4,6	2,76	1,01	4,08	2,51	1,26	16,22	16	24	32	1B	0	5	0	1	0	0	6
2003	3,56	2,3	0,41	4,63	1,61	0,58	13,09	13	19	29	2B	0	4	1	0	0	1	6
2002	3,45	3,56	0,59	3,9	2,27	0,41	14,18	14	23	29	-	0	0	1	0	0	0	1
2001	3,65	1,55	0,88	3,23	2,73	0,78	12,82	11	20	30	-	0	0	0	0	0	0	0
2000	4,1	2,77	0,66	3,18	1,63	1,05	13,39	11	21	30	1B	0	0	0	1	0	0	1
1999	4,3	1,67	1,58	2,81	1,81	1,15	13,32	12	19	28	-	0	0	0	1	0	0	1
1998	3,21	2,74	1,76	3,46	2,93	0,68	14,78	14	24	31	-	0	0	0	0	0	0	0
1997	2,48	3,9	1,78	3,74	3,35	0,82	16,07	15	25	35	-	0	0	0	1	0	0	1
1996	3,18	2,03	2,4	2,12	0,49	2,24	12,46	12	20	28	-	0	0	0	1	0	0	1
1995	5,06	1,71	3,13	4,59	3,41	1,06	18,96	19	29	37	-	1	0	0	0	0	0	1
1994	2,56	5	3,93	3,34	3,22	2,09	20,14	19	30	40	-	1	0	0	0	0	0	1
1993	2,03	1,93	1,13	2,3	3,38	1,82	12,59	11	20	30	-	0	0	0	0	0	0	0
Summary											1S + 11B	40	17	7	12	1	2	79

Note: P – task, B – bronze medal, S – silver medal, G – gold medal

Conclusions

1. The content of the Olympiad tasks is aimed at students demonstrating a creative approach, which includes the presence of a variable range of answers, arguments. The most rigorous requirements are imposed on the answers and task solutions of the participants in the final stage of the Olympiad; therefore, it is necessary to teach them fluency in the mathematical language and the technique of rational reasoning.

2. The analysis of the content of the problems of mathematical Olympiads in the Kyrgyz Republic has shown that the set of tasks contains topics, table 5:

Table 5

Codifier of the main topics of the Olympiad tasks in mathematics

Grade	Algebra	Geometry
7	"Numeric rebuses, parentheses and signs, linguistics. Proportions, shares, percent, concentration. Movement, work, productivity. Logical tasks (truth of statements). Elements of number theory (signs of divisibility, decimal notation of a number). Degree with a natural indicator. Olympiad tricks: dirichlet principle, invariants, colouring, graphs, games; combinatory, weighing"	"Tasks for cutting, gluing, reshaping. Basic geometric shapes. Parallel lines. Adjacent and vertical angles. Signs of equality of triangles. The sum of the angles of a triangle."
8	"Abbreviated multiplication formulas. Transformation of algebraic expressions. Real numbers. The roots Square trinomial. Degree with integer exponent. Graphs of linear and quadratic functions. Hyperbola. Different number systems. Roman numerals. Numerical inequalities. Comparison of numbers. Algebraic inequalities."	"Quadrangle. Parallelogram. Trapezoid. Thales's theorem. Pythagorean theorem. Items of trigonometry. Cartesian coordinates on the plane. Vectors on the plane. Movement. Point and line symmetry. Parallel transfer. Tangent circle. Construction problem. Geometrical locus."
9	"Algebraic transformations. Irrational expressions. Quadratic function and square trinomial. Function graphs. Factorization of algebraic expressions. Search for highs and lows. Proof of inequalities. Vieta formulas for higher degree polynomials. Equations and systems of equations of a higher order. Numeric sequences. Arithmetic and geometric progressions. The method of mathematical induction. Trigonometric expressions and transformations."	"Similar triangles; inscribed and described angles. Tasks for calculating the area."
10 - 11	"Number theory. Graph theory. Game theory. Invariants. Elements of the theory of functions. Elements of game theory. Elements of the theory of optimal control (problems on maxima and minima). Logical tasks. Combinatorics."	"Planimetry. Stereometry."

3. The criteria for evaluating the Olympiad tasks vary depending on the level and status of the Olympiad, the age of its participants. At the same time, when evaluating the tasks of any Olympiad, the independence of students, logical thinking, skills in formulating arguments and novelty of solutions are taken into account. Tasks should form objective and universal educational actions in unity. There is a need for an accurate assessment of the characteristics of the Olympiad problems (the relative complexity of the problem, differentiating ability, the validity of the task, its compliance with the level of training) for a more objective determination of the winners.

4. In the procedure for evaluating the 2019 republican Olympiad, new criteria for assessing the Olympiad tasks were applied: a 10-point assessment of tasks was introduced, other possible ways of solving problems were provided, points were determined for each stage of solving. Compared with the Olympiads of previous years, there was an increase

in the number of winners from the regional regions of Kyrgyzstan, from which it can be concluded that the participation of an independent organization in the methodological guidance, support of the Olympiad and the development of Olympiad tasks contributes to the comprehensible organization, objectivity, and transparency of the Olympiads.

5. Students of the Kyrgyz Republic take part in more than 10 different mathematical Olympiads of international status. Despite the fact that at the Olympiads among the republics of the post-Soviet space, our students show good results, nevertheless their knowledge does not correspond to the Olympiads of the international level. To solve this problem, it is necessary to bring the Olympiads closer to international standards, to develop an Olympiad program on mathematics that will correspond to the level of international mathematical Olympiads, to train professional trainers within the country who can qualitatively prepare schoolchildren for participation in IMO, and to enhance the selection of participants for international Olympiads.

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