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MathARNa: Integrating Augmented Reality and Culturally Responsive STEM in Calculus E-modules to Enhance Students' Critical Thinking Skills

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ABSTRACT

The problem and the aim of the study. Critical thinking is a skill that students must master to keep up with 21st-century learning developments. Critical thinking skills encourage students to develop the ability to make decisions. In higher education, critical thinking skills are crucial in various majors, one of which is the mathematics education major, especially in learning integral calculus. Integrals are one of the important topics because they are widely applied in the real world and subsequent lectures. However, in fact, students' critical thinking skills on the topic of integral mathematics are relatively low. Students' abilities are limited to procedural knowledge, while their conceptual understanding is weak. Therefore, learning is needed that can connect concepts to the real world, which supports students' conceptual understanding, as well as learning innovations that can improve students' critical thinking skills. In line with technological developments, it is necessary to update mathematics learning through the use of ICT-based learning media in the form of e-modules. *The purpose of this study* is to describe the results of the innovation in the development of an e-module for integral calculus based on Augmented Reality (AR) with a Culturally Responsive STEM (CRS) approach to support students' critical thinking skills.

Research methods. This research is a design research development study. This development research went through an iterative cycle by implementing formative evaluation, consisting of the preliminary design and formative evaluation stages. This study involved 37 mathematics education students in the field test. Data collection techniques used were walkthroughs, observations, interviews, questionnaires, documentation, and tests. Data were analyzed based on the research stages. The indicator of success of this research is the resulting product in the form of an AR-based integral calculus e-module with a valid, practical, and effective CRS approach to improve students' critical thinking skills.

Results. The results showed that the MathARNa e-module met the criteria of being very valid (material = 94.8%; media = 97.6%), very practical (91%), and effective in improving students' critical thinking skills ($p < 0.05$). The average critical thinking score increased from 42.32 to 58.59, categorized as high. The integration of AR technology allows students to dynamically visualize integral concepts, while the CRS approach presents a local cultural context that increases learning relevance and engagement. These findings support constructivist and culturally responsive teaching theories that contextual and culturally experience-based learning can improve students' conceptual understanding and higher-order thinking skills.

Conclusion. AR-based integral calculus e-module with a CRS approach can be an effective learning resource for improving students' critical thinking skills. The e-module's context relates to the real world, allowing students to learn integral concepts effectively.

KEYWORDS

e-modules, calculus integral, culturally responsive STEM, augmented reality, critical thinking skills

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INTRODUCTION

Education has long been recognized as a global priority for sustainable human development. The United Nations' 2030 Agenda for Sustainable Development – particularly Sustainable Development Goal 4 (SDG 4) – calls for "inclusive and equitable quality education and lifelong learning opportunities for all," emphasizing not only access but also the development of higher-order competencies such as critical thinking, creativity, and problem solving [1]. UNESCO's Global Education Monitoring Reports consistently highlight that education systems must cultivate learners who can navigate complex digital, cultural, and environmental challenges through analytical reasoning and ethical awareness [2]. Parallelly, the Council of Europe's Reference Framework of Competences for Democratic Culture (RFCDC) underscores the need for education that nurtures reflective thinking, intercultural understanding, and active citizenship [3]. These frameworks, supported by the International Union of Architects' (UIA) Education Commission, stress the synergy between science, technology, culture, and design in shaping sustainable learning environments for the 21st-century. Together, they call for pedagogical transformations that integrate digital innovation and cultural relevance in fostering learners' critical and creative capacities worldwide.

Critical thinking skills are an essential 21st-century competency that plays an important role in shaping students who are able to make rational, reflective, and evidence-based decisions. Facione [4] defines critical thinking as a deliberate and directed reasoning process that includes interpretation, analysis, evaluation, inference, and explanation governed by self-regulation. Ennis [5] emphasizes that critical thinking helps individuals make rational judgments based on valid evidence, whereas Paul dan Elder [6] stated that this ability trains reflection in the thinking process to solve problems logically. In the context of mathematics education, Brookhart, Zohar & Dori explain that critical thinking is the foundation for understanding abstract concepts, analyzing mathematical arguments, and constructing logical proofs [4; 5]. Soboleva & Karavaev [9], Damyanov & Tsankov [10] emphasized that students who have critical thinking skills are more active and creative in learning. Furthermore, Karakozov & Ryzhova, in their research, stated that critical thinking skills can encourage students to make decisions in learning [11]. However, the research results of Dwyer, Hogan, & Stewart show that students' critical thinking skills are still relatively low, especially in mathematics learning, which is abstract and conceptual [12].

Calculus is a fundamental course in higher education, which is the basis for mastering engineering, physics, and economics as described in the work of Tall [13]. Hughes-Hallett states that one of the important topics in calculus is the definite integral, which plays a role in determining area, volume, and cumulative changes in various real phenomena [14]. Students' understanding of the topic of integration is very important, because according to Sealey, integration is widely applied in the real world and subsequent lecture assignments [15].

However, Wagner [16] stated that many students have difficulty understanding the concept of integrals due to their abstract nature and high symbolic complexity. Sealey [15] found that students often failed to connect the formal definition of the Riemann integral with its geometric meaning, Jones [17] identified common misconceptions such as the error of interpreting integrals as simply areas without understanding their relationship to derivatives, while Bezeuidenhout [18] explains that students' knowledge is limited to symbolic procedural knowledge. This condition impacts low critical thinking skills because the learning they experience does not require in-depth analysis, evaluation, and reasoning processes. In Indonesia, a similar problem was found in research by Gunadi et al., who found that calculus is still considered difficult and often problematic [19]. Conventional learning models that focus on procedural exercises hinder the development of critical thinking skills [20]. To overcome these problems, technology-based learning is a strategic alternative to improve understanding of integral calculus concepts.

One of the innovations that aligns with the prominent developments in 21st-century learning is the use of e-modules, due to their ability to support independent, flexible, and interactive learning [21]. Research result Staneviciene & Žekienė [22] confirms that e-modules enable the integration of various multimedia features and systematic learning paths so that students can review the material independently. In line with Adilla et.al [23], Interactive modules containing multimedia (text, audio, images, animations) contribute to self-regulated learning and the regulation of learning rhythm. However, Omer explained that most existing e-modules are still textual and lack dynamic visualizations, thus failing to address conceptual difficulties in calculus [24].

Augmented Reality (AR) technology offers a potential solution by presenting mathematical objects in three-dimensional form that can be visualized and manipulated interactively, based on A survey of Augmented Reality by Azuma [25] and the result of Systematic Literature Review by Akçayır & Akçayır [25]. The results of the meta-analysis by Garzón & Acevedo [26] showed that the use of AR significantly improved conceptual understanding and learning motivation. In the context of mathematics education, Ibáñez and Delgado-Kloos [27] prove that AR strengthens spatial visualization and geometric reasoning skills. In calculus learning, Pinter & Siddiqui [28] explain that AR allows students to understand the geometric meaning of integrals and rotational shapes of objects by linking algebraic expressions to three-dimensional representations. Despite its great potential, the results of the Systematic Literature Review by Lampropoulus found that the application of AR in calculus learning in higher education is still very limited, especially in developing countries [29].

In implementing e-modules, they need to be contextualized to make them more meaningful for students. Garzón & Acevendo [26] suggested that one approach that can be integrated with AR-based e-modules is Science, Technology, Engineering, and Mathematics (STEM). Listiyani, et.al [30] state that the STEM framework can be collaborated with a culture-based approach known as Culturally Responsive STEM (CRS). Ladson-Billing [31] states that the Culturally Responsive STEM (CRS) approach is an innovative strategy that integrates cultural context into science, technology, engineering, and mathematics learning in line with the National Council of Teachers of Mathematics [31]. Hammond [32] explains that CRS helps build cognitive bridges between students' cultural experiences and academic content, thereby encouraging engagement and in-depth understanding. In line with this, the results of the Systematic Review and Research Synthesis [32] show that the CRS approach can increase motivation, problem-solving skills, and critical thinking through culturally relevant learning contexts [33].

In the Indonesian context, the application of CRS is in line with the ethnomathematics paradigm, which studies mathematical ideas embedded in local cultural practices [34]. Research by Rosa & Orey [35] has shown that cultural integration in mathematics learning can improve students' reasoning and conceptual understanding. However, most CRS research still focuses on basic school material and has not been widely applied to advanced topics such as calculus [36]. Therefore, there is a need to develop learning innovations that combine CRS with AR technology so that calculus learning becomes more contextual and interactive.

This research introduces MathARNa (Mathematics Augmented Reality Nusantara), an e-module of integral calculus based on Augmented Reality with a Culturally Responsive STEM (CRS) approach. This innovation links the integral concept with the context of local cultural architecture, namely the K.H. Hasyim Asy'ari Islamic Museum of Nusantara (MINHA), located in Indonesia, which has a distinctive geometric structure, such as a rectangular pyramid that can be modeled mathematically. Within this context, students are invited to link architectural forms with integral concepts, such as the volume of a rotating object and the area of a plane, thereby making conceptual understanding more meaningful.

The success of this research can be seen from previous research, namely; 1) research Sari, Nayazik, & Wahyuni [37] shows that the results of the development of Ethno-STEM based e-modules on the integral volume of rotating objects material meet the eligibility criteria, but the research has not reached the dissemination stage; 2) research Arifin, Pujiatuti, & Sudiana [38] shows that the development of STEM learning media with AR is feasible and effective in improving students' mathematical spatial abilities; and 3) research Winarni, et.al [39] This study shows that AR-based module design for flat-sided geometric shapes is suitable for improving spatial abilities. However, although there has been much research on AR-based e-modules, to date, there has been no research that specifically combines AR and CRS in calculus learning to improve students' critical thinking skills. Most AR research only focuses on visualization and learning motivation [23; 24], whereas CRS research is generally still limited to cultural integration without the support of interactive technology [40]. Thus, there is a research gap in developing e-modules that integrate both approaches synergistically to strengthen students' critical thinking in calculus material.

Thus, the purpose of this study is to describe the development process and assess the validity, practicality, and effectiveness of the Augmented Reality (AR)-based integral calculus e-module with the Culturally Responsive STEM (CRS) approach in improving students' critical thinking skills in integral calculus learning.

RESEARCH METHODS

The type of research used was a design research development study. This development research went through a repetitive cycle according to Plomp & Nieveen, applying formative evaluation [41]. This research procedure was carried out in two stages, namely preliminary and formative evaluation, which includes self-evaluation, expert reviews, and one-to-one (low resistance to revision), small group, and field tests (high resistance to revision) as shown in the following figure.

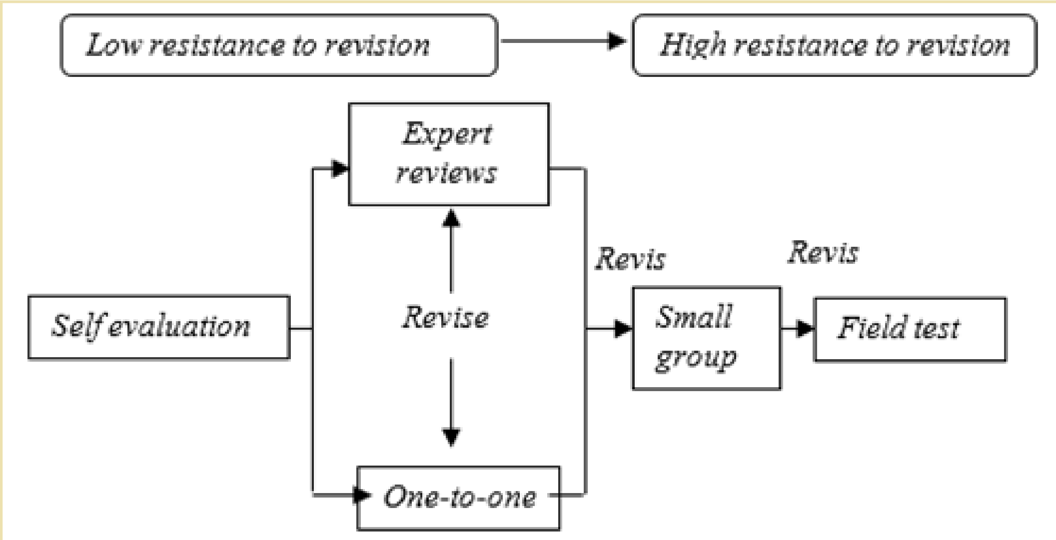


Figure 1 Formative Evaluation Design Flow (source: Tessmer [42]; Zulkardi & Putri [43])

The subjects of this study were three students in the Mathematics Education Study Program at Hasyim Asy'ari University, Jombang, Indonesia, in the one-to-one phase and ten students in the small group phase. The field test involved 37 students from the Mathematics Education Study Program at Hasyim Asy'ari University, Jombang, Indonesia, and Universitas PGRI Jombang, Indonesia, who were currently or had taken integral calculus courses. The purpose of this study was to describe the development process and assess the validity, practicality, and effectiveness of an Augmented Reality (AR)-based integral calculus e-module with a Culturally Responsive STEM (CRS) approach in improving students' critical thinking skills in integral calculus learning.

Data collection techniques used were walkthroughs, observations, interviews, questionnaires, documentation, and tests. Data from the walkthroughs, interviews, questionnaires, documentation, and test results were analyzed based on a formative evaluation cycle. The validity of the developed product was determined by qualitative validation conducted by expert reviews of the entire e-module in terms of content, construct, and language. Quantitative validity was determined by validation of the e-module components. Product practicality was determined by data from one-to-one and small group sessions using questionnaires and interviews. Potential effectiveness criteria were based on field test data based on critical thinking skills indicators.

The data analysis techniques used depended on the research procedure. Data from walkthroughs, documentation, observations, tests, interviews, and questionnaires were analyzed using Tessmer's formative evaluation cycle [42]. Qualitative analysis of validation results for the developed e-module refers to Akbar [44] as follows.

$$P_V = TS_i / TS_{maks} \times 100\%$$

P_V: Validation score percentage

TS_i: Total score obtained

TS_{maks}: Maximum total score

Furthermore, the validity criteria for e-modules obtained from material and media experts are categorized based on the following Table 1.

Table 1 E-module Validity Criteria

Validation Score Percentage	Category
$85,00\%<P_v\leq 100,00\%$	Very Valid
$70,00\%<P_v\leq 85,00\%$	Valid
$50,00\%<P_v\leq 70,00\%$	Less Valid
$01,00\%<P_v\leq 50,00\%$	Not Valid

source: modified by Akbar [44]

Meanwhile, the practicality of the developed e-module was obtained from the results of the questionnaire data analysis at the small group stage. The technical analysis of the data obtained from the expert review questionnaire results refers to Akbar [44] as follows.

$$P_p=TS_i/TS_{maks}\times 100\%$$

P_p: Practicality score percentage
TS_i: Total score obtained
TS_{maks}: Maximum total score

Furthermore, the practicality criteria of the developed e-modules are categorized based on the following Table 2.

Table 2 Practicality Criteria of E-modules

Practicality Score Percentage	Category
$85,00\%<P_p\leq 100,00\%$	Very Practical
$70,00\%<P_p\leq 85,00\%$	Practical
$50,00\%<P_p\leq 70,00\%$	Less Practical
$01,00\%<P_p\leq 50,00\%$	Not Practical

source: modified by Akbar [44]

The effectiveness test was conducted through experimental research with a one group pre-test-post-test design, as shown in Table 3.

Table 3 Design Pre-test and Post-test in One Group

Pre-test	Treatment	Post-test
T_1	X	T_2

The t-test calculation uses a paired sample t-test with the help of SPSS. The researcher hypothesizes that the developed e-module is effective in improving students' critical thinking skills in integral calculus material at a significant level $\alpha=5\%$. The researcher's hypothesis is accepted if the calculation of the value is significant $<\alpha$, whereas if the calculation of significant values $\geq\alpha$ then the researcher's hypothesis is rejected.

RESEARCH RESULTS

The development of an e-module for integral calculus using a CRS approach based on Augmented Reality (AR) emphasizes the iterative cycle according to Plomp & Nieveen. The results of the development of the MathARNa e-module are presented as follows.

1 Preliminary Stage

In the preliminary stage, an analysis of needs, concepts and materials was carried out as a basis for designing an AR-based integral calculus e-module with a CRS approach.

Needs Analysis

A needs analysis was conducted to identify real-world problems in integral calculus learning, which served as the basis for developing the e-module. Data was obtained through interviews with lecturers teaching Integral Calculus, lecture observations, and questionnaires conducted with first-semester students in the Mathematics Education study program.

The analysis revealed that integral calculus learning in the classroom still focuses on mechanistic procedures and does not facilitate a deep understanding of the concepts. Students tend to memorize solution steps without connecting the integral process to geometric meaning or real-life applications. Most students also experience difficulty visualizing the area and volume of solids of revolution due to the lack of dynamic and interactive learning media.

Furthermore, the learning tools used are still predominantly text and mathematical symbols, thus not supporting the development of critical thinking skills that require analysis, evaluation, and interpretation. Based on the survey, 87% of students stated that they need learning media that can display 3D visual representations and relate integral material to real-world phenomena and cultural elements.

From the educator's perspective, the lecturers stated the need to develop e-modules that can: 1) present integral concepts visually and interactively, 2) integrate local cultural values and contexts as characteristics of the CRS approach, 3) direct students to analyze and interpret results, not just calculate.

Concept Analysis

Concept analysis was conducted to identify the structure and hierarchical relationships between concepts within the integral topic. This analysis served to ensure that the developed e-module presented a logical learning flow and was appropriate to the students' cognitive level.

The analysis results indicated that the integral concept has the following hierarchical structure.

Table 4 Integral Concept Mapping

Concept Level	Description	Relationship with Other Concepts
Prerequisite	Understanding functions, limits, derivatives, and graphical representation of functions.	It is the basis for understanding the meaning of anti-derivatives and the relationship between change and accumulation.
Basic Concept	Indefinite integral as anti-derivative, constant of integration, and basic integral operations.	Becoming the foundation for the concept of definite integrals.
Intermediate Concept	The definite integral as the area under a curve, the fundamental theorem of calculus, and its relation to the Riemannian limit.	Be a bridge between basic concepts and integral applications in real contexts.
Applied Concepts	Application of integrals to calculate the area of plane regions and volume of rotating objects.	Realizing the relationship between integrals and geometric aspects and physical phenomena.

This analysis shows that students' understanding still tends to be procedurally oriented, and they are unable to connect mathematical symbols with geometric meanings and application contexts. Therefore, the developed e-module is aimed at strengthening multiple representation skills (symbolic, numeric, graphic, and verbal) and providing exploratory experiences through interactive AR visualizations.

Material Concept

The results of the material analysis obtained a relationship between integrals and cultural contexts that allows the application of the CRS approach, where mathematical concepts are explained through cultural phenomena such as MINHA's architectural design.

Table 5 Mapping of Material in E-Modules

Sub-material	Main Concept	Student Difficulties	E-Module Development Solutions (AR + CRS)	Facilitated Critical Thinking Aspects
Indefinite Integral	Integrals as anti-derivatives, constants of integration, and the relationship between functions and their derivatives.	Students tend to memorize formulas without understanding the meaning of anti-derivatives; often ignore the constant of integration.	Presenting interactive visualization of the relationship between function graphs and their derivatives using AR; local cultural context such as architecture of stairs of MINHA to visualize continuous functions.	Interpretation, Analysis
Definite Integral	The concept of area under a curve, the fundamental theorem of calculus, and the relationship with the Riemannian limit.	Students do not yet understand the geometric meaning of integrals and have difficulty connecting the Riemann limit with the area of a region.	AR simulations show the formation of the area of a Riemann rectangle and the relationship with the integral result; local contexts such as measuring the area of MINHA museum roof.	Inference, Explanation
Integral Applications in Area and Region of a Plane	Calculation of the area between two curves, boundaries of regions, and interpretation of graphs.	Difficulty determining curve boundaries and drawing integrated regions; poor spatial understanding.	The AR feature displays the interaction of two curves in 3D; CRS contexts such as calculating the area of MINHA museum roof.	Analysis, Evaluation
Integral Applications in Volume of Rotating Objects	The concept of rotation of a plane about an axis, the method of discs, rings and cylindrical shells.	Students have difficulty distinguishing methods, visualizing the results of rotation, and understanding the relationship between area and volume.	AR displays a simulation of the rotation of an area into a 3D shape; integration of cultural context such as the shape MINHA museum pillars to explain the concept of the volume of a rotating object.	Explanation, Reasoning
STEM Synthesis and Connections	Integration of integral concepts with aspects of science, technology, engineering, and culture.	Students see integrals simply as a mathematical procedure without cross-disciplinary connections.	Project-based exploratory activities: students design physical or digital models of cultural objects (e.g., stairs, roof, or pillars of museum) based on mathematical functions and calculate their volumes using integrals.	Interpretation, Analysis, Evaluation

E-Module Design

The initial design phase was conducted after the results of the needs analysis, concept analysis, and material analysis were obtained. The goal was to develop an initial design (prototype I) for an integral calculus e-module that combines Augmented Reality (AR) technology and a CRS approach to enhance students' critical thinking skills. The design process was carried out by considering the suitability between material characteristics, user needs, and interactive learning design principles.

a. Structure of e-module content

The e-module structure is designed to follow a constructivist-based learning flow, which consists of six main components as shown in Table 6 below.

Table 6 E-Module Content Structure

Part	Content and Activity Description	AR / CRS Features	Relation to Critical Thinking
1. Contextual Introduction	Introduction to the importance of integration in real life and in the context of local culture.	AR-based introductory video display featuring local cultural objects and their mathematical forms.	Interpretation and Engagement (understanding the context and conceptual meaning).
2. Basic Concept of Indefinite Integrals	Explanation of the meaning of anti-derivative and constant of integration through function graph exploration activities.	AR simulation: the relationship between a function and its derivative is displayed in 3D; the user can change the slope of the function.	Analysis (analysing the function-derivative relationship).
3. Definite Integrals and Geometric Meaning	Presentation of the concept of area under a curve using the Riemann Integral approach.	AR displays the area formation from small rectangles that refine the partition to approach the actual area.	Inference and Explanation (concluding symbolic-geometric relationships).
4. Application of Integral in Area of a Plane	The activity of calculating the area between two curves through graphical observation and numerical calculations.	AR displays two curves and the area between them in 3D space with the context of MINHA's roof architectural design.	Analysis and Evaluation (assessing visual and calculation results).
5. Application of Integrals in Volume of Rotating Objects	Virtual experiments on rotation of regions about the X/Y axis using three methods (disk, ring, cylindrical shell).	AR displays a simulation of curve rotation into 3D shapes such as traditional jugs, gongs, and vases.	Reasoning and Explanation (explaining the relationship between the area and volume resulting from rotation).
6. Critical Reflection and Evaluation	Students answer reflective questions based on analysis and interpretation of simulation results..	Integration of interactive quizzes and CRS case studies, for example calculating the volume of a local mosque dome.	Evaluation and Self-Regulation (evaluating results, interpreting data, and drawing conclusions).

The display design of the AR-based integral calculus e-module that integrates the CRS approach can be seen as follows.

b. Visual design and technological features

The e-module was developed by combining text, images, animation, audio, and AR markers. Users simply scan the markers using a mobile device to display 3D objects such as curves, planes, and rotating bodies. Some of the main features designed include: 1) interactive 3D objects: students can rotate, enlarge, or reduce the 3D model of the integral result, and 2) dynamic visualization: changes in function values in real-time affect the shape of the resulting curve and volume.

Figure 2 E-module Design

Figure 3 MathARNa application display on Android

The display design is aligned with the principles of a simple user experience, intuitive navigation, and soft colors that facilitate user focus. Each e-module page also features a "scan image" link that activates an interactive visualization mode.

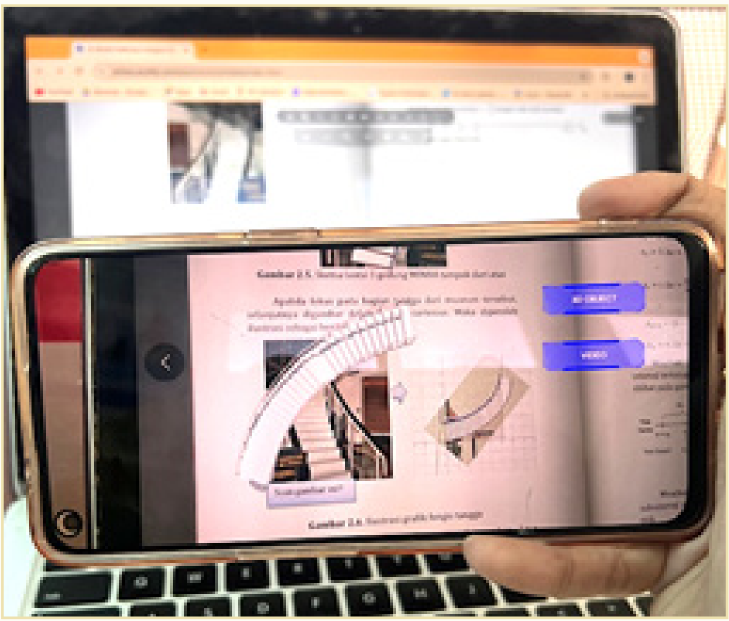


Figure 4 Scan the image in the e-module

2. Formative Evaluation Stage

Self-Evaluation

At this stage, a review of the initial prototype design was conducted while preparing research data collection instruments related to the development process. The review was conducted by checking the suitability of the e-module design with the material concept in terms of material feasibility (e-module substance), learning media (instructional), and language. In addition, a review was also conducted on the media aspect, including aspects of language, graphics, and media accessibility. The designed e-module was given to colleagues for validation to obtain prototype 1 (initial prototype).

Expert Reviews and One-to-one

At this stage, the initial prototype that has been developed is then given to expert reviewers and three students (one-to-one) simultaneously (parallel). The validation results from each expert validator can be seen in the following table.

Table 7 Material Expert Validation Results

Measured Aspects	Average Score	Percentage (%)	Category
Material Eligibility (e-Module Substance)	4,88	97,6	Very Valid
Instructional Media	4,56	91,2	Very Valid
Language	4,80	96,0	Very Valid
Average	4,74	94,8	Very Valid

Table 8 Media Expert Validation Results

Measured Aspects	Average Score	Percentage (%)	Category
Language	5,00	100	Very Valid
Graphics	4,75	95,0	Very Valid
Media accessibility	4,90	98,0	Very Valid
Average	4,88	97,6	Very Valid

Based on Tables 7 and 8, the validity of the developed integral calculus e-module using the CRS approach met the validity criteria for both material and media, reaching an average of 94.8% and 97.6%, respectively, categorized as very valid.

Table 9 Expert Review and One-to-One Advice and Comments from Media Aspects

Validation	Suggestions/Comments	Revision
Expert Review	There are some images that are displayed less clearly.	Change the image resolution to make it clearer.
	Instructions for using AR applications need to be clarified.	Added instructions on how to install and run the AR application to scan images in the e-module.
	The image quality in the video is low, it is better to increase the resolution.	Change the image resolution in a video.
	If possible, you can add images or videos related to the related practice questions to make it interesting.	Add pictures and videos to practice questions related to the museum context.

One-to-one	R-1	The image display in AR is a bit unclear.	Replacing an unclear image.
	R-2	Audio intonation in video is too fast.	Change the audio intonation in a video to make it slower.
	R-3	Figure 2.5 is difficult to scan.	Replace image 2.5 with a clearer and brighter image so it is easy to scan.

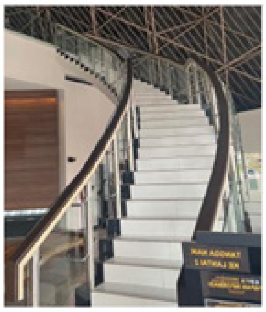
Prototype 1 was administered to three students (one-to-one) outside the research subjects to assess the e-module's readability, suitability to the material taught, ease of access, and use of augmented reality. At this stage, the researchers also solicited suggestions and comments from media and materials experts to revise prototype 1. Several suggestions and comments from the experts and students in the one-to-one subjects were then used as the basis for revisions to prototype 1. These suggestions and comments are outlined in the table 9.

Table 10 Expert Review and One-to-One Suggestions and Comments on Material Aspects

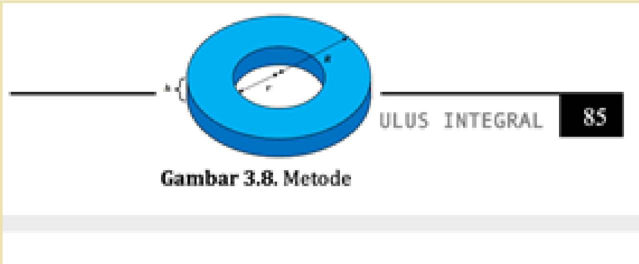
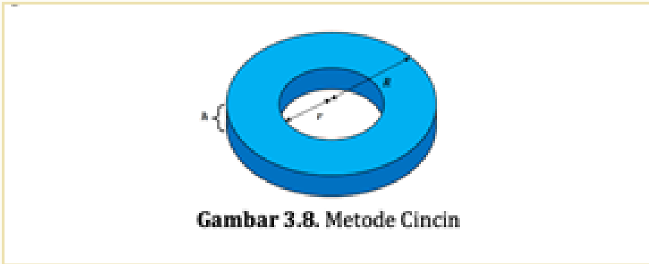
Validation		Suggestions/Comments	Revision
Expert Review		There is a typo. Please check again.	Recheck the contents of the e-module and correct any misspelled words.
		There is an error in writing ∫ (cos2x)/((sin2x)^3) dx	Change the spelling to ∫ (cos2x)/(sin2x)^{3}dx
		There are variables and symbols that are inconsistent in their writing.	Recheck the formulas in the e-module and correct any inappropriate variables and symbols.
One-to-one	R-1	• Example 1 in chapter 2 has no equal sign in the sigma limit. • On page 28, the image does not appear and is less clear. • The image on page 50 does not appear when scanned.	• Fix formula writing. • Replace image on page 28. • Replace image on page 50
	R-2	• Example question number 6 in Chapter 2 is confusing. • On page 78, the video intro is too long and the animation is not fast enough.	• Clarify the information in question no.6 by adding related function formulas presented for the front or back roof of the museum. • Shorten the intro and animation on page 78.
	R-3	Example question number 1 in Chapter 3 requires a graphic image.	Adding a function graph of the area whose volume will be calculated in Chapter 3, question number 1.

Based on input and suggestions from expert review and one-to-one in Table 9 and 10 regarding materials and media, revisions were made to prototype 1. Some of these revisions include:

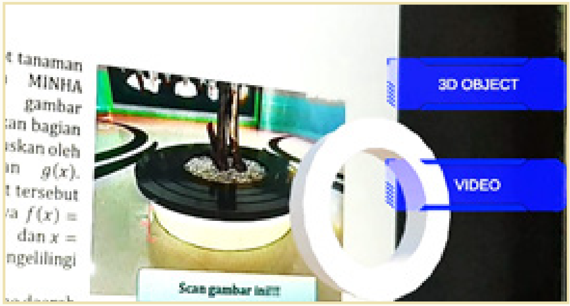
1) Replace image 2.5 on the e-module.

Before revision	After revision
 **Gambar 2.5.** Sketsa lantai 1 gedung MINHA tampak dari atas	 **Gambar 2.5.** Sketsa lantai 1 gedung MINHA tampak dari atas

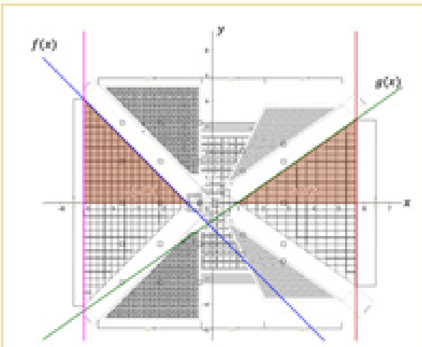
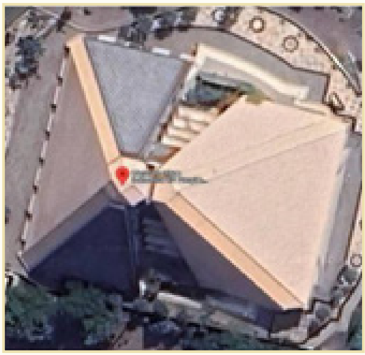
2) Fixed some image positions on the e-module.

Before revision	After revision
	

3) Adding 3D objects to the sample questions in Chapter 3

Before revision	After revision
	

4) Replacing image 2.15 (top view of MINHA roof)

Before revision	After revision
	

Some input from the validator and input at the one-to-one stage were used as material to revise the e-module and AR application. The results of the revision at this stage produced prototype 2, which was then tested at the small group stage.

Small Group

At the small group stage, a set of AR-based e-modules was tested on 10 mathematics education students who had taken integral calculus courses. After the small group process was completed, interviews were conducted with the small group subjects. From the interview results, it can be concluded, among others:

1. The AR-based e-module that was developed is feasible to be used or implemented in integral calculus learning.
2. Students are very interested and enthusiastic about using AR-based e-modules in integral calculus learning.

3. The AR-based e-module is very effective because it can be used anytime and anywhere as long as it is within internet reach.
4. The AR-based e-module that was developed can be used as an independent learning medium outside the classroom.
5. The AR-based e-module that was developed is very interesting and good.
6. The ethno-STEM context, in the form of exploring the MINHA museum, is very interesting in supporting the understanding of the material in the e-module.

In the small group stage, a questionnaire was administered to lecturers and students after implementing integral calculus learning to determine students' responses to the use of AR-based e-modules with the CRS approach. The questionnaire results were used to measure the practicality of the developed e-modules. The average practicality results of the e-modules are presented in the following table.

Table 11 Results of the practicality questionnaire

Measured Aspects	Average Score	Percentage (%)	Category
Usable	4,55	91,0	Very Practical
Easy to use	4,55	91,0	Very Practical
Appealing	4,64	92,8	Very Practical
Efficient	4,45	89,0	Very Practical
Average	4,55	91,0	Very Practical

Based on the data analysis results from the small group stage, the AR-based integral calculus e-module developed with the CRS approach met practical criteria. The revised results were then piloted on field test subjects.

Field Test

Field test data were used to assess the effectiveness of the AR-based integral calculus e-module with the CRS approach on students' critical thinking skills in the integral calculus course. This field test phase involved the following analysis results using SPSS.

Table 12 Paired Sample T-test Results

Information	Average Value
Average pre-test score	42,32
Average post-test score	58,59
Sig value (2-tailed)	0,000

Based on Table 12, it can be seen that the sig value (2-tailed) is smaller than the significance value α that is 0,000<0,05. Therefore, the researcher's hypothesis is accepted. This means that the AR-based integral calculus e-module with the CRS approach effectively improves students' critical thinking skills. Furthermore, descriptively, there is a difference in the average critical thinking skills of students before and after being taught using the AR-based e-module with the CRS approach, namely from 42.32 to 58.59. This indicates that students' thinking skills have increased significantly.

DISCUSSION

The results of the study indicate that the Augmented Reality (AR)-based integral calculus e-module with the Culturally Responsive STEM (CRS) approach effectively improves students' critical thinking skills. This effectiveness is evident from the significant increase in the average post-test score compared to the pre-test and the results of the paired sample t-test with a significance value <0.05. These results strengthen the theory that the use of interactive technology in learning can improve higher-order thinking skills, as explained by Ibáñez & Delgado-Kloos [27] and Garzón & Acevedo [26]. The use of AR can strengthen conceptual representations and increase students' cognitive engagement in the science and mathematics learning process.

From the perspective of constructivist learning theory, the findings of this study support the views of Piaget and Vygotsky, who emphasize that knowledge is constructed through active interactions between individuals and their environment. In this context, AR allows students to spatially explore integral concepts, as explained by Pinter & Siddiqui [28], AR technology is capable of visualizing three-dimensional mathematical objects, enabling students to understand the symbolic, geometric, and conceptual relationships of definite and indefinite integrals.

Furthermore, the results of this study also support the findings of the study. Listiyani et al. [30] and Rosa & Orey [35], that the integration of cultural context in mathematics learning through the CRS approach can increase students' motivation, reflection, and conceptual connections. In the MathARNa e-module, the Nusantara cultural context, such as the architecture of the K.H. Hasyim Asy'ari Nusantara Islamic Museum (MINHA), is used to bridge the abstract concept of integrals with real geometric forms. This is in line with the view Hammond [32] which explains that the Culturally Responsive Teaching approach serves as a cognitive bridge between students' cultural experiences and academic concepts.

From a critical thinking theory perspective, the results of this study support the framework proposed by Facione and Ennis, who emphasize that critical thinking encompasses the abilities of interpretation, analysis, inference, explanation, and evaluation, which can be enhanced through exploration- and reflection-based learning activities [1; 42]. In the MathARNa e-module, AR-based learning activities encourage students to analyze the relationship between functions and their derivatives, draw inferences from visual representations of Riemann integrals, and evaluate the results of object rotation simulations. These activities align with the critical thinking dimensions of Brookhart and Dwyer et al., which emphasize the importance of reflection and reasoning in understanding abstract mathematical concepts [5; 9].

Thus, the findings of this study support previous theories and demonstrate coherence with related research in educational technology and mathematics education. This study also provides a theoretical contribution in the form of a synthesis between Augmented Reality (AR) and Culturally Responsive STEM (CRS), which has rarely been integrated into calculus learning in higher education [29]. The development of AR-based e-modules with a cultural context can be an innovative model for teaching abstract mathematics material, increasing learning engagement, and fostering students' critical thinking skills in a meaningful way.

CONCLUSION

This study concludes that the Augmented Reality-based integral calculus e-module with the CRS approach is effective in improving students' critical thinking skills. The developed e-module was found to be highly valid in terms of content and media, very practical for both lecturers and students, and statistically proven effective ($\alpha < 0.05$). Improved students' critical thinking skills encompassed all Facione indicators: interpretation, analysis, inference, explanation, and evaluation.

Theoretically, the results of this study support constructivist learning theory and critical thinking theory, and reinforce previous findings regarding the effectiveness of AR and the CRS approach in enriching mathematics learning. Practically, this e-module has implications for increasing interactivity and meaningful learning, making calculus more contextual, visual, and engaging. Pedagogically, the integration of AR technology with the local cultural context bridges the gap between abstract concepts and the real world, encouraging collaboration, reflection, and higher-order thinking skills in students.

Further research is recommended to apply this e-module to other mathematics topics and measure its impact on other variables such as students' spatial abilities, problem-solving, and digital literacy.

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REFERENCES

1. UNESCO. (2016). *Education 2030: Incheon Declaration and Framework for Action Towards Inclusive and Equitable Quality Education and Lifelong Learning for All*. UNESCO.
2. UNESCO. (2021). *Global Education Monitoring Report 2021/2: Non-state actors in education*. UNESCO.
3. Aceska, N., Barrett, M., & Tibbitts, F. (2025). Reference Framework og Cometences for Demicratic Culture (RFCDC): Guidance Document for Education for Sustainable Development. Council of Europe.
4. Facione, P. a. (2011). Critical Thinking : What It Is and Why It Counts. *Insight assessment,* 1–28.
5. Ennis, R. H. (1996). Critical Thinking Dispositions: Their Nature and Assessability. *Informal Logic, 18*(2), 165–182. https://doi.org/10.22329/il.v18i2.2378
6. Rihard Paul, L. E. (2006). Critical Thinking The Miniature Guide tools. In *Developments in Business Simulation and Experiential Learning: Proceedings of the Annual ABSEL conference* (Vol. 33).
7. Zohar, B., & Dori, Y. (2012). *Higher Order Thinking in Science Education*. Springer.
8. Brookhart, S. M. (1918). Your Classroom. In *Journal of Education* (Vol. 88, Nomor 18). https://doi.org/10.1177/002205741808801819
9. Soboleva, E. V., & Karavaev, N. L. (2020). Characteristics of the project-based teamwork in the case of developing a smart application in a digital educational environment. *European Journal of Contemporary Education, 9*(2), 417–433. https://doi.org/10.13187/ejced.2020.2.417
10. Damyanov, I., & Tsankov, N. (2018). The role of infographics for the development of skills for cognitive modeling in education. *International Journal of Emerging Technologies in Learning, 13*(1), 82–92. https://doi.org/10.3991/ijet.v13i01.7541
11. Karakozov, S. D., & Ryzhova, N. I. (2019). Information and education systems in the context of digitalization of education. *Journal of Siberian Federal University - Humanities and Social Sciences, 12*(9), 1635–1647. https://doi.org/10.17516/1997-1370-0485
12. Dwyer, C. P., Hogan, M. J., & Stewart, I. (2014). An integrated critical thinking framework for the 21st century. *Thinking Skills and Creativity, 12*, 43–52. https://doi.org/10.1016/j.tsc.2013.12.004
13. Tall, M. (2013). How Humans Learn to Think Mathematically.
14. Hughes-Hallett, D., McCallum, W. G., & Gleason, A. M. (2017). *Calculus Single Variable* (7 ed.). Wiley.
15. Sealey, V. (2014). A Framework for Characterizing Student Understanding of Riemann Sums and Definite Integrals. *Journal of Mathematical Behavior, 33*, 230–245. https://doi.org/10.1016/j.jmathb.2013.12.002
16. Wagner, J. F. (2018). Students' Obstacles to Using Riemann Sum Interpretations of the Definite Integral. *International Journal of Research in Undergraduate Mathematics Education*, 4, 327–356. https://doi.org/https://doi.org/10.1007/s40753-017-0060-7
17. Jones, S. R. (2014). *The Frequencies of Various Interpretations of the Definite Integral in a General Student Population*, 3, 401–408.
18. Bezuidenhout, J., & Olivier, A. (2000). Students' conceptions of the integral. In *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education*, 2, 73–80.
19. Schulz, A. (2024). Assessing student teachers' procedural fluency and strategic competence in operating and mathematizing with natural and rational numbers. *Journal of Mathematics Teacher Education, 27*(6), 981–1008. https://doi.org/10.1007/s10857-023-09590-7
20. Saraswati, S., & Rodliyah, I. (2020). Blended Learning Berbasis Edmodo: Proses Pengembangan pada Mata Kuliah Kalkulus Dasar. *Jurnal Gantang, 5*(2), 133–142. https://doi.org/10.31629/jg.v5i2.2415
21. Hasanati, A., Supriana, E., & Mufti, N. (2023). Effectiveness of Digital Modules with Recitation Programs to Improve Students' Conceptual Understanding and Critical Thinking Skills in Learning Work and Energy. *The 5th International Conference on Mathematics and Science Eucation (ICoMSE) 2021: Science and Mathematics Education Research: Current Challenges and Opportunities*, 050015. https://doi.org/https://

doi.org/10.1063/5.0114390
22. Staneviciene, E., & Žekienė, G. (2025). The Use of Multimedia in the Teaching and Learning Process of Higher Education: A Systematic Review. *Sustainability, 17*(19), 8859. https://doi.org/10.3390/su17198859
23. Adilla, P., Pada, A. U. T., Muhammad, N., Sarong, M. A., & Artika, W. (2024). *Use of Interactive Learning Modules to Improve Student Learning Achievement and Self-Regulated Learning*. Atlantis Press SARL. https://doi.org/10.2991/978-2-38476-216-3_24
24. St Omer, S. M., Evers, K., Wang, C. Y., & Chen, S. (2025). Technology-enhanced mathematics learning: review of the interactions between technological attributes and aspects of mathematics education from 2013 to 2022. *Humanities and Social Sciences Communications, 12*(1), 1–13. https://doi.org/10.1057/s41599-025-05475-7
25. Azuma, R. T. (1997). A survey of augmented reality. *Presence: Teleoperators and Virtual Environments, 6*(4), 355–385.
26. Garzón, J., & Acevedo, J. (2019). Meta-analysis of the impact of Augmented Reality on students' learning gains. *Educational Research Review, 27*(April 2018), 244–260. https://doi.org/10.1016/j.edurev.2019.04.001
27. Ibáñez, M. B., & Delgado-Kloos, C. (2018). Augmented reality for STEM learning: A systematic review. *Computers and Education, 123*, 109–123. https://doi.org/10.1016/j.compedu.2018.05.002
28. Pinter, L., & Siddiqui, M. F. H. (2024). Enhancing Calculus Learning through Interactive VR and AR Technologies: A Study on Immersive Educational Tools. *Multimodal Technologies and Interaction, 8*(3). https://doi.org/10.3390/mti8030019
29. Cevikbas, M., Bulut, N., & Kaiser, G. (2023). Exploring the Benefits and Drawbacks of AR and VR Technologies for Learners of Mathematics: Recent Developments. *Systems, 11*(5). https://doi.org/10.3390/systems11050244
30. Listiyani, L. R., Wilujeng, I., Suyanto, S., & Pratama, D. H. (2025). EthnoSTEM-based Learning Tools: Connecting Cultural Heritage with STEM Education. *International Electronic Journal of Elementary Education, 17*(4), 593–609. https://doi.org/10.26822/iejee.2025.402
31. NCTM. (2014). Principles to Actions - Ensuring Mathematics Success fo All.
32. Hammond, Z. (2015). Culturally Responsive Teaching & The Brain: Promoting Authentic Engagement and Rigor Among Culturally and Linguistically Diverse Students. In *Educacao e Sociedade* (Vol. 1, Nomor 1). SAGE Publications, Inc.
33. Nugraha, M. G., Kidman, G., & Tan, H. (2023). Pre-service teacher in STEM education: An integrative review and mapping of the Indonesian research literature. *Eurasia Journal of Mathematics, Science and Technology Education, 19*(5). https://doi.org/10.29333/ejmste/13155
34. D'Ambrosio, U. (1985). Ethnomathematics and Its Place in the History and Pedagogy of Mathematics. *For the Learning of Mathematics, 5*(1), 44–48.
35. Rosa, M., & Orey, D. (2016). Humanizing Mathematics through Ethnomodelling. *Journal of Humanistic Mathematics, 6*(2), 3–22. https://doi.org/10.5642/jhummath.201602.03
36. Cheng, M. M., Lacaste, A. V., Saranza, C., & Chuang, H. H. (2021). Culturally responsive teaching in technology-supported learning environments in marine education for sustainable development. *Sustainability (Switzerland), 13*(24). https://doi.org/10.3390/su132413922
37. Sari, N. R., Nayazik, A., & Wahyuni, A. (2022). Pengembangan E-Modul Berbasis Ethno-STEM Pada Materi Volume Benda Putar Integral. *JNPM (Jurnal Nasional Pendidikan Matematika), 6*(3), 565–579. https://doi.org/10.33603/jnpm.v6i3.7289
38. Arifin, A. M., Pujiastuti, H., & Sudiana, R. (2020). Pengembangan media pembelajaran STEM dengan augmented reality untuk meningkatkan kemampuan spasial matematis siswa. *Jurnal Riset Pendidikan Matematika, 7*(1), 59–73. https://doi.org/10.21831/jrpm.v7i1.32135
39. Winarni, S., Fadhila, N. S., Kumalasari, A., Marlina, M., & Rohati, R. (2023). Desain Modul Berbasis Augmented Reality dalam Meningkatkan Kemampuan Spasial Siswa SMP. *Jurnal Cendekia: Jurnal Pendidikan Matematika, 7*(3), 3321–3337. https://doi.org/10.31004/cendekia.v7i3.2654
40. Fitri, H. S. A., Putri, E., & Chuchai, S. (2025). Trends in Culturally Responsive Teaching for Science EducationL A Bibliometric Analysis. *Journal of Technological Pedagogy and Educational Developmen, 1*(2), 36–52.
41. Plomp, T., & Nieveen, N. (2007). An Introduction to Educational Design Research. The Netherlands.
42. Tessmer, M. (1993). *Planning and Conducting Formative Evaluations: Improving the Quality of Education and Training*. Kogan Page.
43. Zulkardi, & Putri, R. I. I. (2006). *Mendesain Sendiri Soal Kontekstual Matematika*. Prosiding Konferensi Nasional Matematika XIII. Semarang: IndoMS.
44. Akbar, S. (2013). *Instrument Perangkat Pembelajaran* (PT Remaja Rosdakarya (ed.))
45. Ennis, R. H. (2011). *The Nature of Critical Thinking: An Outline of Critical Thinking Dispositions*.

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